Learning Deterrence vs. Encouragement: Optimal Pricing and Return Policy

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Abstract

Information about a certain product is easy to acquire nowadays and the uninformed buyer can choose the optimal level of information acquisition (before-trading and post-trading) after observing the selling mechanism committed by the seller. A selling mechanism contains a selling price and a return policy that specifies the return probability and the amount of refund. Different selling mechanisms are used to induce different levels of buyer's private learning. We find that the seller's optimal selling mechanism either encourages full learning (with Free Return) or deters the buyer from private learning (with Learning Deterrence), while Stochastic Return mechanism that induces partial learning is always sub-optimal. Trading is efficient for both sufficiently optimistic and pessimistic buyer. Though the seller's expected revenue is monotone on the buyer's prior belief, the buyer's ex-ante trading surplus is non-monotone. More surprisingly, the buyer is worse off under a Free Return mechanism. When cost of learning converges to zero, the result converges to the standard niche-mass market analysis.

1 Introduction

Information about a certain product is easy to obtain nowadays. Before trading, consumer can search for product information, customer reviews and professional evaluations on the internet. They can also go to the off-line store to try the product for more concrete user experience. In addition, consumers often take product's return policy into account before making purchase decisionIt is conceivable that consumers are more likely to make purchase decision when they can freely return, since even they eventually find it a bad match after purchase, they always have the option to return it. Commonly, if the product is not returnable, consumers tend to make more cautious decision and conduct learning before they purchase it. Considering this, return policy plays an important role in shaping consumer's learning behaviour.

In general, return policy varies across platforms and product categories. Free return (full refund) and no return (non-refundable payment) are the most common return policies. Apart from these, airline companies usually charge a fixed fee for ticket refund and the fees vary across different travel class and the timing of cancellation. Fashion platform like Farfetch offers free return and free home collection, while others, like Luisavaroma, offers free return as long as the customers agree to exchange the same amount of refund into store credit, otherwise they need to pay a fixed return cost. More surprisingly, Amazon offers refund with no return, however it is random at the moment of purchasing and the company specifies it as "Amazon *may* determine that a refund can be issued without requiring a return". Considering all these different return policies, in this paper, we use a reduced form-the chance of return and the amount of refund, to capture all return policies. We define *Free Return* as certain return with full refund and *No Return* represents non-refundable/returnable policy, while *Stochastic Return* is of non-degenerate chance of return with some particular refund.

To make the model more concrete, we consider a seller (she) selling one unit of indivisible product to a single buyer (he), who is uncertain about the product's matching value to him (the matching value is either high or low). We use to denote the buyer's belief of having high matching value and his prior belief $_0$ is commonly known. There is no production cost and no return cost. The seller commits to a persistent selling mechanism that specifies the selling price and a return policy. After the buyer observes the selling mechanism, he makes the decisions on when and how much to learn about the matching value; whether and when to purchase/return the product. In particular, the buyer can acquire information before he purchases the item (*before-trading learning*) and decides whether to buy as information comes in. If yes, he can perform *post-trading learning* and decide whether to keep the item or request a return. Buyer's learning outcomes are unobservable to the seller. For simplicity, we assume the learning process follows exponential distribution that high value generates signal with rate and learning incurs a flow cost k. Furthermore, we assume the buyer's learning process is the same before and after trading. The main question of this paper is to find the optimal selling mechanism for the seller.

Different from the standard approach in mechanism design literature,[1] we consider for

[1]First, solve the buyer's optimal learning dynamic for each feasible selling mechanism. Then calculate the seller's expected revenue conditional on the selling mechanism and buyer's best response

each feasible buyer's learning dynamic (i.e., the distribution of stopping beliefs), what is the most profitable selling mechanism that induces such learning dynamic. The result shows that any selling mechanism that provides the buyer with surplus different from his continuation payoff $V^0(\ )$ under the "No Return" mechanism with same selling price is weakly dominated (Lemma 1). Loosely speaking, suppose the buyer's surplus from post-trading learning is extracted (by the proposed return policy) below a level such that he would rather conduct all learning before purchase, then he will never request the return. On the other hand, if the return policy rewards the buyer from performing post-trading learning (providing the buyer with surplus strictly larger than $V^0(\ )$), the seller can always take away part of the reward (decrease the buyer's continuation payoff from post-trading learning) and gain larger expected revenue.

Regarding a "No Return" mechanism, the buyer would perform all learning before trade and stop learning till either 1) a signal (high matching value) is realized and he buys it; or 2) his posterior gets sufficiently low and he quits the market. For a fixed selling price, the buyer's optimal stopping problem is well-studied in the literature (Wald (1947), Keller, Rady, and Cripps (2005)). In this paper, however, the selling price is endogenously chosen by the seller and it affects the level of $V^0(\ )$. Therefore, Proposition 1 summarizes all possible buyer's learning dynamics and his payoffs $V^0(\ )$ given the selling price. Lemma 2 explicitly pins down the return policy as a function of the selling price and the stopping belief. We then frame the problem as an information design problem that the seller maximizes her expected revenue (transfer) over all feasible distributions of buyer's stopping beliefs. Before we elaborate the results, it would be useful to introduce three types of selling mechanism: (1) Learning Deterrence; (2) Free Return; and (3) Stochastic Return.

Learning Deterrence is a special case of "No Return" mechanism. It refers to selling a *non-refundable* item at the price that makes buyer indifferent between purchasing the item immediately with no future opportunity to return (later we call it consuming the item) and exerting before-trading learning. To break the tie, we let the buyer purchase the item immediately and hence trading is efficient. Note that in order to prevent the buyer from private learning, the seller has to compensate the buyer with surplus (weakly) larger than his continuation payoff from performing learning. Therefore, the selling price of Learning Deterrence has to be attractive enough and it turns out to be the lowest among these three mechanisms. However, the seller guarantees herself safe revenue equals the selling price.

Free Return allows the buyer to return the item and get full refund anytime after his purchasing. Therefore, the buyer performs learning after he purchases the item. Once the matching value is realized to be high, he consumes the item. When his

to it. Finally find the revenue maximizing selling mechanism.

expected matching value is getting sufficiently low, he returns it with full refund. In this case, trading is inefficient and the probability of ex-post return depends on the seller's choice of selling price. Under Free Return, the buyer is able to perform ***full learning*** in the sense that he stops learning either with a realized high matching value or the continuation payoff from keeping learning becomes zero. The selling price is the highest among these three mechanisms but there is no safe revenue (the seller gets zero revenue if the buyer returns the item).

Stochastic Return is to set a selling price and the item is required to be returned with some probability meanwhile the refund is of a fixed amount. For some given price, the return policy is designed to induce some particular level of ***partial learning*** in the sense that the buyer optimally stops learning even though the continuation payoff for keep learning is positive. In particular, when the buyer requests a return, the seller allows him to keep the item with positive probability, which implies positive trading surplus upon return. In this way, the seller can provide the buyer with positive return payoff meanwhile ensure herself a positive return revenue. With Stochastic Return, the selling price is higher than Learning Deterrence and lower than Free Return, but the seller can guarantee herself positive safe revenue.

For different prior beliefs, the seller optimally chooses different types of selling mechanism. However, Theorem 1 shows that Stochastic Return is always sub-optimal. To show this, we go through two steps. (1) Firstly, for each selling price, we find the (locally) optimal stopping belief that the seller would like to guide the buyer to stop. Essentially, the seller faces a trade-off between receiving more return transfer (if inducing a more optimistic stopping belief) and reimbursing less often (if inducing a more pessimistic stopping belief). Moreover, with a lower selling price, the refund that the seller commits to pay to induce the same stopping belief turns out to be smaller. Therefore, she becomes less sensitive to the return rate and optimally induces a larger stopping belief to guarantee higher return transfer. In other words, the seller endogenously adapts a higher stopping belief to the reduction of selling price. (2) Given this monotonicity, to find out the optimal Stochastic Return mechanism is equivalent to pick the optimal level among all locally optimal stopping beliefs. Now the seller balances between larger safe revenue (by inducing larger locally optimal stopping belief) and larger expected "bonus" upon the realization of a signal (by the opposite). It turns out that the first incentive dominates the second one if the seller intends to enforce larger stopping belief, while the opposite holds if she wants to enforce smaller stopping belief. In other words, the seller's expected revenue is quasi-convex in the (locally optimal) stopping belief, which implies partial learning is never optimal.

Hence, the optimal selling mechanism is either Learning Deterrence that induces no learning or Free Return that induces full learning. To select one of these two types of selling mechanism, the seller is essentially comparing low price but larger probability of sale with high price but low probability of sale. Theorem 2 shows that when the buyer's prior belief belongs to the closed interval $[\underline{\mu}^F, \overline{\mu}^F] \subset (\underline{\mu}^L, \overline{\mu}^L)$, the seller optimally chooses a Free Return mechanism.[2] Otherwise, for $\mu_0 \in [\underline{\mu}^L, \underline{\mu}^F) \cup (\overline{\mu}^F, \overline{\mu}^L]$ she optimally chooses Learning Deterrence. The interval $[\underline{\mu}^L, \overline{\mu}^L]$ denotes the set of prior belief that learning is feasible,[3] therefore the seller's optimal choice over how much learning to induce is only relevant in this set. To illustrate the optimality between Learning Deterrence and Free Return, when prior is low, there is less chance that a high matching value is realized and hence larger chance of ex-post return, therefore the seller is willing to sacrifice the price in order to achieve efficient trading. Therefore, for relevantly pessimistic buyer, Learning Deterrence is optimal. When the prior is relatively high, even with Learning Deterrence, the seller is able to charge the buyer a relatively higher price and therefore sacrificing the probability of sale to increase the price a little bit is not optimal, which implies the optimality of Learning Deterrence for relatively optimistic buyer. For moderate prior belief $\mu_0 \in [\underline{\mu}^F, \overline{\mu}^F]$, the seller optimally selects Free Return.

With the optimal selling mechanism, the seller's expected revenue is increasing in the buyer's prior belief. However, the buyer's ex-ante trading surplus is not monotone in his prior. Firstly, consider Learning Deterrence only, to deter the buyer's private learning, the seller has to compensate him with at least his private benefit from performing learning. With a less informative prior belief, the buyer potentially gains larger benefit from learning. Therefore, even with Learning Deterrence, the buyer's ex-ante surplus from trading is non-monotone. When his prior belongs to the region where the seller optimally offers a Free Return mechanism, the total trading surplus is smaller compared to Learning Deterrence since trading is not efficient and there is a sunk cost for learning. Meanwhile, the selling price is higher and the buyer obtains even less trading surplus, i.e., there is a discontinuous reduction of buyer's trading surplus at the end points of $[\underline{\mu}^F, \overline{\mu}^F]$. Furthermore, when the learning is more efficient, i.e., $\frac{k}{\lambda}$ is smaller, the set of prior belief that the seller optimally selects Free Return expands. When $\frac{k}{\lambda}$ converges to zero, the buyer can almost learn perfect information. Therefore, with Free Return, the seller sets the price arbitrarily close to v_h and let go the buyer almost sure to have low matching value. With Learning Deterrence, she sets price arbitrarily close to v_l. This converges to the standard analysis of niche-mass market. The ratio $\frac{v_l}{v_h}$ pins down the cutoff of prior such that if $\mu_0 > \frac{v_l}{v_h}$ the seller only

[2] It is possible that $[\underline{\mu}^F, \overline{\mu}^F]$ does not exist. For illustration, we consider the situation that it exists.

[3] By learning is feasible, we mean if μ_0 belongs to this set, there exists a selling price such that the buyer's private learning is feasible. Commonly, the buyer tends to exert learning when the selling price is high. But for the buyer whose prior belief is sufficiently informative, *no* selling mechanism can induce him to learn, therefore the seller optimally sets a non-refundable price same as the prior mean.

sells to high-value buyer and charges the high price, otherwise, the seller covers the whole market and charges a low price.

Related literature. There is a growing literature on mechanism design incorporating information as part of optimal choice. Considering price discrimination, Li and Shi (2017) allow the seller to disclose different additional information for different types of buyer. They show that partial and discriminatory disclosure weakly dominates full disclosure in terms of seller's revenue. Guo, Li, and Shi (2020) then characterize the property of optimal discriminatory disclosure. Bergemann and Pesendorfer (2007) allow the buyer to acquire information but the information accuracy is controlled by the seller. In stead of letting the seller optimally make restrictions on the buyer's learning process, Roesler and Szentes (2017) allow the buyer to acquire fully flexible and costless information, anticipating that the seller's pricing decision depends on his expected information outcomes. They identify that the buyer-optimal information structure generates a unit-elastic demand and the seller is indifferent between charging any price and therefore choose the lowest price. Ravid, Roesler, and Szentes (2019) discuss the same scenario but the seller and buyer move simultaneously. They characterize the set of equilibria when information is free and show that the equilibrium converges to the worst free-learning equilibrium when the learning cost vanishes.

Our paper stays in the line of mechanism design when the buyer can endogenously acquire information (Shi (2012) and Mensch (2020)), meaning that the seller uses different mechanisms to control the buyer's optimal information acquisition. Shi (2012) assumes rotational-ordered information technology and shows the optimality of posted price in the case of single buyer. Moreover, the optimal price, conditional on a fixed information choice being the equilibrium outcome, is smaller than the monopoly price when this information choice is exogenous. This is because the price increment affects the buyer's incentive to acquire information and hence changes the trading probability, which turns out to lower the seller's expected revenue. However, in our paper, by assuming dynamic information acquisition, it is actually the return policy that pins down the buyer's optimal stopping and the seller can sustain the same trading probability when price increases.

Mensch (2020) discusses the same question with Shi (2012), but allows flexible information acquisition with cost as the expected difference in a posterior-separable measure of uncertainty. It characterizes the set of all implementable mechanisms (*contour mechanisms*), which consist of triplets of allocation probabilities, prices and beliefs. Therefore, the problem can be translated into Bayesian persuasion. Such translation is also adopted in our model. However, we choose instead exponential bandit as information technology and assume additive time cost, therefore the cost for the same Blackwell experiment will be the same for different prior beliefs, which is not true for exible information.[4] This allows us to discuss how the buyer's prior belief (ex-ante informativeness) a ects the seller's optimal selling mechanism and in turn his own trading surplus.

Many other papers also discuss seller's pricing strategy when the buyer can acquire information dynamically (Bonatti (2011), Bergemann and Valimaki (2000) and Bergemann and Valimaki (1996)). The most related papers are Lang (2019) and Pease (2018). Both consider the seller's optimal pricing when the buyer can sequentially acquire information before purchasing. Therefore, the buyer's optimal stopping only responds to the selling price. However, in our paper, by introducing the return policy, the seller can induce more exible stopping time and obtain non-negative transfer upon the buyer's stopping. In Lang (2019), buyer's information acquisition follows Brownian motion whose drift depends on the buyer's true valuation, which is normally distributed. He shows that the trading probability is increasing in the buyer's prior valuation (mean). However, we show that trading e ciency is non-monotone in buyer's prior belief. The major di erence is that, in our model, the buyer's prior belief (probability of having high valuation) captures his initial informativeness. Hence as long as the buyer is su ciently optimistic or pessimistic, the potential gain from acquire information becomes small, which give rise to e cient trading. While in Lang (2019), the buyer's prior mean represents his ex-ante valuation of the product, which does not a ect the informativeness of the buyer's prior belief.

The remainder of this paper is organized as follows: Section 2 discusses the model, Section 3 characterizes the buyer's payo s from all "Nor Return" mechanisms, Section 4 discusses Learning Deterrence, Section 5 studies Learning encouragement, and Section 6 solves the optimal selling mechanism. Section 7 concludes.

2 Model

There is one seller selling one unit of indivisible item to a single buyer. The product values zero to the seller and there is no cost of production and return. The buyer is initially uninformed about his matching value with the product. Matching value is either high v_h or low v_l, $v_h > v_l > 0$. Let be the buyer's belief of having high matching value and we call this as the buyer's type. Type- buyer's expected matching value follows $E(v|\ ) = \ v_h + (1 \ \ )v_l$. The buyer's initial type $_0$ is common knowledge and his posterior belief evolves over time conditional on his learning dynamic, which will

[4] There does not exist a uni ed measure of uncertainty, regardless of the prior belief, that can represent the additive time cost of Poisson signal (See Appendix A of Mensch (2020) and Pomatto, Strack, and Tamuz (2019)).

be speci ed later. To di erentiate, we use to denote the time and (= 0) = $_0$. The seller commits to a selling mechanism, which speci es (1) a selling price $t_b > 0$ which is the transfer made from buyer to the seller at the time of purchasing[5] and (2) a return policy that contains: a) the probability that the buyer is required to return the item at the time of the buyer requesting a return and b) the refund he can gets regardless of whether a return is required.[6] Equivalently, we can use $(x_r; t_r)$ to denote the return policy.[7] x_r 2 $[0; 1]$ is the probability that a buyer keeps the item after requesting a return and t_r 2 $[0; t_b]$ is the net transfer made from buyer to seller at the time of requesting a return.[8] Overall, one typical selling mechanism is characterized by f $t_b; (x_r; t_r)$g. Given this, a "No Return/Refund" mechanism can be written down as f $t_b; (1; t_b)$g. "Free Return" can be represented as f $t_b; (0; 0)$g. It is without loss to focus on selling mechanism that type-1 buyer purchases the item without requesting further return (i.e., v_h $t_b > v_h x_r$ t_r). This is formally proved in the Appendix and the reader can take this as an assumption. For simplicity, we assume both parties have no discount. The buyer's outside option is normalized to be zero.

A type- buyer's payo is realized when he consumes the item.[9] And if so, he cannot request a return regardless of the return policy. In particular, he obtains payo E(vj) t_b if he purchases a non-refundable item and consumes it right away, while he obtains utility E(vj)x_r t_r if he requests a return at policy $(x_r; t_r)$. Let B = 0 if the buyer does not purchase the item up to time and B = 1 otherwise. Let R = 0 if the buyer does not request a return up to time and R = 1 otherwise. Denote $_r$ = min f : R = 1g. Naturally, we have the feasibility constraint that $_r$ minf : B = 1g. The seller's revenue is denoted as.

$$= E \int_0^1 t_b dB + (t_r \quad t_b) dR \tag{1}$$

After observing the selling mechanism, the buyer can decide (1) whether to learn and if yes, when to learn-i.e., acquiring information before purchasing the item or after; and (2a) while performing before-trading learning, when to stop learning and whether to

[5] The reader may think of that, the seller can o er a pair of $(x_b; t_b)$, which speci es the probability of sale x_b and the transfer t_b at the time of purchasing. We show that it is without loss to focus on $x_b = 1$ at the end of the Appendix.

[6] It refers to the Amazon example of "refund without requiring a return". Besides, in some cases, the refund could depend on whether the buyer returns the item. Due to quasi-linear payo structure, it is without loss to focus on the expected refund.

[7] In general, the seller can design a vector of return policies to screen di erent types of buyers. It is without loss to focus on one piece of return policy.

[8] To understand it, if the buyer requests a return given return policy $(x_r; t_r)$, then the seller issues a refund t_b t_r to the buyer and at the meantime she rolls a dice. With probability 1 x_r, the buyer has to return the item to the seller, while with probability x_r, he can still keep it.

[9] The reader can consider that consuming the item provides free and perfect information about the matching value.

purchase the item upon stopping, (2b) while performing post-trading learning, when to stop and whether to request return upon stopping. In this paper, we focus on Poisson learning with additive ow cost. If the buyer decides to learn, he needs to pay a ow cost k, and signals arrives at Poisson rate if his true matching value is high. Conditional on learning, the buyer's belief updates regardless of the timing that learning is performed.

$$^{0}(\ ) = \quad (\ )(1 \quad (\ )) \ < \ 0$$

However, the subtle di erence between before-trading learning and post-trading learning is that: with before-trading learning, the buyer can always quit the market with payo zero, whereas if he decides to learn after trading, then opt-out may not be an option if the seller does not allow "Free Return". Denote $v_{hb} = v_h \quad t_b$ as the consumer surplus that a high matching value buyer gets when he purchases the item without further return. We can write down the Bellman equation for post-trading learning:

$$V_P(\ (\ )) = \max f \, E(vj \ (\ )) \quad t_b; E(vj \ (\ )) x_r \quad t_r;$$
$$kd \ + \ (\ ) d \ v_{hb} + (1 \quad (\ ) d \ ) V_P(\ (\ + d \ )) g$$

The rst term of the right-hand side denotes type- ( ) buyer's payo from consuming the item right away. The second term is his payo from requesting a return. The third term represents his continuation payo at belief ( ): ( ) d is the probability of obtaining a signal within time d , while ( ) d is the probability of obtaining no signal, conditional on which, the buyer's belief moves to (+ d) and the corresponding valuation is $V_P(\ (\ + d \ ))$. The Bellman equation for before-trading learning:

$$V_B(\ (\ )) = \max f \, 0; V_P(\ (\ )); \ kd \ + \ (\ ) d \ v_{hb} + (1 \quad (\ ) d \ ) V_B(\ (\ + d \ )) g$$

Note that, performing learning before purchasing the item ensures that, the buyer can always decide to quit the market (getting payo 0) or switch to post-trading learning (getting payo $V_P(\ (\ ))$ by purchasing the item. If type- ( ) buyer continues learning, we have the similar term of continuation payo as post-trading learning.

It is useful to introduce the buyer's optimal learning dynamic given a "No Return" policy. Note that with a "No Return" policy $f \, t_b; (1; t_b) g$, the buyer performs learning before trading,[10] and therefore his continuation payo $V^0(\ )$ is speci ed as following:

$$V^0(\ (\ )) = \max f \, 0; E(vj \ (\ )) \quad t_b; \ kd \ + v_{hb} \ (\ ) d \ + (1 \quad (\ ) d \ ) V^0(\ (\ + d \ )) g$$

In this paper, we only focus on Markov strategy. No matter the learning is conducted before-trading or post-trading, as long as the buyer keeps learning, we can have the following di erential equation:[11]

$$(1 \quad) \ V \ ^{0}(\ ) + \ V \ (\ ) = \ v_{hb} \quad k \qquad \text{(ODE)}$$

[10] With a "No Return" policy, after the buyer purchases the item, his payo is linear in belief. Therefore post-trading learning is not necessary, $V_P(\ (\ )) = E(vj \ (\ )) \quad t_b$.

[11] Whose general solution is denoted as $V(\ ) = v_{hb} \quad \frac{k}{}[1 + (1 \quad) \log(\frac{}{1})] + (1 \quad) C$.

Note that $V_B(\mu) \geq V_P(\mu)$ since purchasing the item and then performing post-trading learning is always an option when the buyer performs before-trading learning. Therefore, if $V_B(\mu) > V_P(\mu)$ for some μ, then type-μ buyer optimally chooses before-trading learning and $V_B(\mu) = V^0(\mu)$. Furthermore, since the buyer can always disregard the return policy (treating all selling mechanism as "No Return"), $V_B(\mu) \geq V^0(\mu)$ for all μ. Since the buyer always stops if his belief jumps to 1, to simplify exposition, later on, when we say stopping belief, we mean the non-degenerate stopping belief.

Lemma 1. For any given selling price t_b and all optimal selling mechanism,

$$\max\{V_B(\mu); V_P(\mu)\} = V^0(\mu); \ \forall\mu:$$

To see the intuition, suppose the seller intends to set an aggressive return policy, with which if the buyer purchases the item and stops post-trading learning at belief μ, he obtains payoff $V_P(\mu) < V_B(\mu) = V^0(\mu)$. Then a rational buyer would instead not purchase the item and perform all learning before-trading. Therefore, the return policy will not be realized and the seller gets the same expected revenue as if she sets a "No Return" mechanism with the same selling price. The above condition holds anyway. Now consider the seller sets an attractive return policy intending to reward the buyer from purchasing the item. With which, the buyer purchases the item at some point and while he stops post-trading learning at belief μ and requests a return, he gets payoff $V_P(\mu) > V^0(\mu)$. We can then calculate the transfer the seller obtains upon the buyer's optimal stopping at μ, which equals to the allocation value minus the buyer's surplus:[12]

$$t_r = E(v|\mu)x_r - V_P(\mu) = E(v|\mu)\frac{V_P'(\mu)}{v_h - v_l} - V_P(\mu)$$

Recall that conditional on learning, the ODE holds regardless of the return policy. Hence for a fixed μ, if $V_P(\mu)$ is decreasing, $V_P'(\mu)$ is increasing. Therefore the seller can decrease $V_P(\mu)$ and adjust x_r to induce the same stopping belief μ. At the meantime, she obtains larger return transfer, which implies larger expected revenue.[13] Contradiction of optimality. Hence, for all optimal selling mechanisms, the above condition holds. Obviously, the next step is to pin down $V^0(\mu)$.

[12] For μ to be an incentive compatible stopping belief, the smooth pasting condition should be satisfied, $V_P'(\mu) = d[E(v|\mu)x_r - t_r]/d\mu = (v_h - v_l)x_r$. This implies the second equality.

[13] By inducing the same stopping belief, the ex-ante probabilities of return and successful sale are the same.

3 Characterization of $V^0(\ )$

In this section, we study the buyer's learning behaviour given a "No Return" contract $f\, t_b; (1; t_b) g$. Since the ODE is a ected by the type-1 buyer's surplus $v_{hb} = v_h \quad t_b$, which is part of the selling mechanism chosen by the seller, therefore we start to use $V^0(\ ; v_{hb})$ in stead of $V^0(\ )$. Recall that with a "No Return" contract, the buyer performs all learning before purchasing the item. Following the standard result, for a xed v_{hb}, we can simply pins down the buyer optimal stopping belief $(v_{hb}) = \frac{k}{v_{hb}}$,[14] which is decreasing in v_{hb}. Intuitively, when the high matching value buyer's surplus v_{hb} is reduced, performing learning in order to wait for a good signal is less attractive for the buyer and he optimally stops learning at a larger belief if no signal arrives. With the boundary condition $V(\hat{\ }) = 0$, we can then pin down the ODE's solution, which is also the buyer's continuation payo for learning, denoted as $V(\ ; v_{hb})$.

$$V(\ ; v_{hb}) = \frac{1}{\ }[\quad k + \quad v_{hb} \quad k(1 \quad) \log(\frac{\ = (1 \quad)}{k = (\ v_{hb} \quad k)})]$$

However, to ensure that learning is necessary, there is an implicit assumption: given v_{hb}, when the buyer stops learning at $\hat{\ }(v_{hb}) = \frac{k}{v_{hb}}$, he indeed prefers to quit the market rather than accepting the price. For any xed v_{hb}, this constraints is just the buyer's ex-post IR. However, here we call it the Learning Feasibility constraint, since it pins down the feasible set of buyer's prior belief that there exists some v_{hb}, given which the buyer's private learning is feasible.

$$E(v j \hat{\ }(v_{hb})) \quad (v_h \quad v_{hb}) \quad 0 \tag{LF}$$

To avoid trivial result, we assume there exists two distinct roots $\underline{v}^L_{hb} < \bar{v}^L_{hb}$ that this constraint binds.[15] Let $\underline{\ }^L = \hat{\ }(\bar{v}^L_{hb}) < \bar{\ }^L = \hat{\ }(\underline{v}^L_{hb})$.[16] In particular, if the buyer is endowed with su ciently informative prior (i.e., $_0 > \bar{\ }^L$ and $_0 < \underline{\ }^L$), there does not exists a v_{hb} with which the buyer is willing to learn. To illustrate, take $_0 < \underline{\ }^L$ as example. When the buyer's prior is su ciently pessimistic (less chance of obtaining a signal), if seller wishes to encourage buyer to learn, he has to lower down the price and provide su ciently large surplus v_{hb}. However, at such an inexpensive price, buyer would have no incentive to learn but to purchase the item. As a result, there is no selling mechanism can induce buyer with su ciently low prior to learn.

To introduce one more notation, let $\bar{\ }(v_{hb}) = f \quad$: equation (2) holds g. That is, type-$\bar{\ }(v_{hb})$ buyer is indi erent between consuming the item at price $v_h \quad v_{hb}$ or performing learning and obtaining the continuation payo from it.

$$V(\ ; v_{hb}) = E(v j \ ) \quad (v_h \quad v_{hb}) \tag{2}$$

[14] By $V(\hat{\ }) = 0$ and $V^0(\hat{\ }) = 0$, we can pin down $\hat{\ } = \frac{k}{v_{hb}}$ and $C = \quad v_{hb} + \frac{k}{\ } \log(\frac{k}{v_{hb} \quad k})$.

[15] $\underline{v}^L_{hb} = \frac{v_h \quad v_l}{2} \quad \frac{1}{2}\sqrt{(v_h \quad v_l)(\quad 4k = \quad + v_h \quad v_l)}$ and $\bar{v}^L_{hb} = \frac{v_h \quad v_l}{2} + \frac{1}{2}\sqrt{(v_h \quad v_l)(\quad 4k = \quad + v_h \quad v_l)}$

[16] It is interesting to note that $\underline{\ }^L = 1 \quad \bar{\ }^L$. To see this, the binding (LF) can be reduced to $(1 \quad) = \frac{k}{(v_h \quad v_l)}$ by substituting $v_{hb} = \frac{k}{\ }$.

Proposition 1. (Optimal buyer learning given $\{t_b; (1; t_b)\}$)

(1) If $v_{hb} < \underline{v}^L_{hb}$ or $v_{hb} > \overline{v}^L_{hb}$, the buyer optimally chooses not to learn and

$$V^0(\ ; v_{hb}) = \max\{0; E(v|\ )\quad(v_h\quad v_{hb})\}$$

(2) When $v_{hb} \in [\underline{v}^L_{hb}; \overline{v}^L_{hb}]$, if $_0 < \hat{\ }(v_{hb})$ or $_0\quad \bar{\ }(v_{hb})$, the buyer does not learn and

$$V^0(\ ; v_{hb}) = \max\{0; E(v|\ )\quad(v_h\quad v_{hb})\}$$

If $_0 \in [\hat{\ }(v_{hb}); \bar{\ }(v_{hb}))$, the buyer exerts learning and optimally stops till either a signal arrives or belief falls below $\hat{\ }(v_{hb})$.

$$V^0(\ ; v_{hb}) = \begin{cases} 0; & < \hat{\ }(v_{hb}) \\ V(\ ; v_{hb}); & \hat{\ }(v_{hb}) \quad < \bar{\ }(v_{hb}) \\ E(v|\ )\quad(v_h\quad v_{hb}); & \quad \bar{\ }(v_{hb}) \end{cases}$$

Figure 1 is a graphical illustration of Proposition 1. As mentioned above, for sufficiently high $v_{hb} > \overline{v}^L_{hb}$ and sufficiently low $v_{hb} < \underline{v}^L_{hb}$, the Learning Feasibility constraint does not hold. That is, with such v_{hb}, the buyer optimally chooses not to learn regardless of his prior. Intuitively, with sufficiently high v_{hb}, the selling price is small enough so that performing learning to avoid purchasing the item at low matching value is not desirable. While if v_{hb} is sufficiently low, even a high value buyer obtains little trading surplus and hence performing learning in order to realize high matching value is less attractive. With moderate $v_{hb} \in [\underline{v}^L_{hb}; \overline{v}^L_{hb}]$, the standard results of exponential bandits apply. There exists two cutoffs, $\hat{\ }(v_{hb})\quad \bar{\ }(v_{hb})$, that characterize the buyer's optimal learning behaviour. Essentially, if the buyer's prior falls into $[\hat{\ }(v_{hb}); \bar{\ }(v_{hb})]$, he (weakly) prefers to exert learning till his belief falls below $\hat{\ }(v_{hb})$ and he quits the market or a good news arrives and he decides to buy. Otherwise, he does not learn. Note that the measure of this set $[\hat{\ }(v_{hb}); \bar{\ }(v_{hb})]$ varies across v_{hb}.

Figure 1: Optimal learning dynamic with a "No Return" contract

Proposition 1 basically characterizes the buyer's optimal learning dynamic, taking v_{hb} (or the selling price) as an exogenous variable. However, our main purpose is to analyze the seller's endogenous choice of the selling mechanism (including the price). Therefore, it would be useful to consider the inverse version. Let $\bar{v}_{hb}(\mu) = \bar{\mu}^{-1}(\mu)$ be the inverse function of $\mu(v_{hb})$ with domain $[\underline{\mu}^L; \bar{\mu}^L]$. That is, if the seller sets a non-refundable price $v_h - \bar{v}_{hb}(\mu_0)$, a type-$\mu_0$ buyer is just indifferent between performing learning and accepting the price. Hence, if the seller charges lower price (or provides type-1 buyer with surplus $v_{hb} \geq \bar{v}_{hb}(\mu_0)$), the buyer optimally chooses not to learn and purchases the item immediately. In other words, with $v_{hb} \geq \bar{v}_{hb}(\mu_0)$, the seller can deter the buyer's private learning. Conversely, if the seller charges a higher price ($v_{hb} < \bar{v}_{hb}(\mu_0)$), a type-$\mu_0$ buyer strictly prefers to learn until the price rises above $v_h - \frac{k}{\mu_0}$ and the buyer prefers to quit the market. To conclude, with $v_{hb} \in [\frac{k}{\mu_0}; \bar{v}_{hb}(\mu_0)]$, the seller can enforce the buyer to exert private learning.

Denote $\tilde{\mu}(\mu_0) = \hat{\mu}(\bar{v}_{hb}(\mu_0))$. This captures type-$\mu_0$ buyer's optimal stopping belief if $v_{hb} = \bar{v}_{hb}(\mu_0)$, which is also his smallest stopping belief that the seller can induce. The set $[\tilde{\mu}(\mu_0); \mu_0]$ then captures the all possible type-μ_0 buyer's stopping beliefs that are inducible. From Figure 2, we can see the measure of this interval, $\mu_0 - \tilde{\mu}(\mu_0)$, is non-monotonic in μ_0. It increases from 0 at $\mu_0 = \underline{\mu}^L$ and then decreases to 0 at $\mu_0 = \bar{\mu}^L$ (see Claim 3 in the Appendix), which confirms that learning is feasible for the buyer with prior $\mu_0 \in [\underline{\mu}^L; \bar{\mu}^L]$.

Proposition 2. When $\mu_0 < \underline{\mu}^L$ and $\mu_0 > \bar{\mu}^L$, The optimal selling mechanism is $\{E(v|\mu_0); (1; E(v|\mu_0))\}$. The trading happens with probability 1 and the seller captures all trading surplus.

Proof. Given Lemma 1, for all optimal selling mechanisms $V_B(\mu_0; v_{hb}) = V^0(\mu_0; v_{hb})$. Endowed with sufficiently pessimistic or optimistic prior, the buyer obtains payoff $V_B(\mu_0; v_h - t_b) = \max\{0; E(v|\mu) - t_b\}$. Suppose the seller offers a "No Return" mechanism with price $t_b = E(v|\mu)$. Then the buyer weakly prefers to purchase the item and the seller can extract all trading surplus. To verify that this selling mechanism $\{E(v|\mu_0); (1; E(v|\mu_0))\}$ is optimal, note that the seller's profit equals to the total trading surplus minus the surplus that buyer obtains from the trade. In this case, the trading surplus is the largest since trading happens with probability one and the posterior mean of matching value equals to the prior mean for all possible learning dynamic, meanwhile the buyer gets zero trading surplus. □

4 Deterring Learning

In this section, we discuss the revenue maximizing selling mechanism that can deter the buyer's private learning. Besides, we assume the buyer breaks the indifference in terms of the seller's favor. In particular, if the seller sets $v_{hb} = v_{hb}(\mu_0)$, then we assume the buyer purchasing the item immediately without learning.[17]

Proposition 3. (Learning deterrence) For $\mu_0 \in [\mu_L, \bar{\mu}_L]$ and conditional on $v_{hb} \geq v_{hb}(\mu_0)$, it is optimal to set a "No Return" contract: $\{t(\mu_0); (1; t(\mu_0))\}$, where $t(\mu_0) = v_h - v_{hb}(\mu_0)$. Trading is efficient and the buyer purchases the item without learning.

Proof. Notice that when $v_{hb} \geq v_{hb}(\mu_0)$, Lemma 1 and Proposition 1 imply that

$$V_B(\mu_0; v_{hb}) = V_P(\mu_0; v_{hb}) = E(v|\mu_0) - (v_h - v_{hb}):$$

Therefore the trading is efficient. The seller's revenue then equals the total trading surplus $E(v|\mu_0)$ minus the surplus provided to the buyer $V_B(\mu_0; v_{hb})$, which is just the selling price $v_h - v_{hb}$. Since $v_{hb} \geq v_{hb}(\mu_0)$, it is optimal to let the inequality bind and hence $t(\mu_0) = v_h - v_{hb}(\mu_0)$. □

To interpret this proposition, to deter buyer's private learning, the seller has to lower the price so that accepting the price is more attractive for the buyer compared to learning. $t(\mu_0) = v_h - v_{hb}(\mu_0)$ is the highest achievable price to prevent type-μ_0 buyer from learning. To see this, suppose the seller increases this price by some small number, then the buyer obtains trading surplus $v_{hb} - \epsilon < v_{hb}(\mu_0)$ after observing a good news. The continuation payoff for keep learning is then reduced but the reduction is less than the price increment.[18] Therefore the buyer would strictly prefer to learn and the seller cannot prevent the buyer's private learning by setting a price larger than $t(\mu_0)$. We call the selling mechanism $\{t(\mu_0); (1; t(\mu_0))\}$ as *Learning Deterrence*. With which, the buyer obtains the same payoff from purchasing the item immediately and exerting before-trading learning till a signal arrives and he purchases the item, or quitting the market if his belief keeps falling down. As shown in Figure 2, $V_B(\mu; v_{hb}(\mu_0))$ crosses $E(v|\mu) - t(\mu)$ at μ_0. Essentially, $E(v|\mu_0) - t(\mu_0) = V(\mu_0; v_{hb}(\mu_0))$. We let the buyer break the indifference by purchasing the item immediately, therefore trading is efficient.

It would be interesting to note that, with $v_{hb}(\mu_0)$, if type-μ_0 buyer chooses to learn, he performs the longest learning and stops at the smallest inducible belief $\underline{\mu}(\mu_0)$. Hence, $V(\mu_0; v_{hb}(\mu_0))$ is the highest continuation payoff that type-μ_0 buyer can ever get from

[17] If the buyer does not learn, the trading happens with probability 1 the transfer is just the selling price which is higher than all feasible transfer upon return (Otherwise, requesting return is dominated by consuming the item for all types of buyer).

[18] Because the probability of realizing a signal and accepting the price is less than 1.

learning. To prevent the buyer's from private learning and ensure not-to-learn is indeed the buyer's best response, such "Learning Deterrence" mechanism compensates the buyer with the largest potential gain he could obtain from private learning and $V(\ _0; v_{hb}(\ _0))$ is the buyer's trading surplus. Note that $V(\ _0; v_{hb}(\ _0))$ is non-monotone in the buyer's prior belief (shown in Figure 2). Intuitively, for more uncertain prior belief, the buyer enjoys larger private bene t if he exerts learning, therefore to deter his private learning, the seller has to provide him larger surplus. Since trading happens with probability one and no learning cost is incurred, the highest total trading surplus, which is just the buyer's expected value $E(v|\ _0)$, is achieved (Referring to Figure 2, the two dashed arrow intervals are of the same length). Furthermore, the seller's revenue just equals the selling price $t(\ _0)$, which is increasing in $_0$.

Figure 2: Learning deterrence

Proposition 4. (Properties of seller's pro t and buyer's rent for learning deterrence)
(1) The buyer's surplus $V(\ _0; v_{hb}(\ _0))$ is supported on $[\underline{\ }^L; \overline{\ }^L]$. It equals zero at the two boundary points and it increases rst and then decreases.
(2) The seller's pro t $t(\ _0)$ is supported on $[\underline{\ }^L; \overline{\ }^L]$ and increasing in $_0$. $t(\ ) \quad E(v|\ )$ and the equality holds if $_0$ is either $\underline{\ }^L$ or $\overline{\ }^L$.

5 Encouraging Learning

For $_0 \in [\underline{\ }^L; \overline{\ }^L]$, the seller can enforce the buyer to learn by providing type-1 buyer with the right amount of surplus, $v_{hb} \in [\frac{k}{_0}; \overline{v}_{hb}(\ _0)]$. Given Lemma 1, for all optimal selling mechanism, $V_B(\ ; v_{hb}) = V^0(\ ; v_{hb}) \quad V_P(\ ; v_{hb})$ for all $\in [0;1]$. In other words, the buyer can always apply merely before-trading learning to obtain the valuation $V^0(\ ; v_{hb})$. However, in this case, there is no realized return request on the equilibrium path, meaning that the seller cannot gain by having the choice to design the return policy. This issue is driven by how the buyer is assumed to break indifference. Following the assumption in the literature, we let the buyer breaks the tie in favor of the seller. In particular, if $V_B(\ ) = V_P(\ )$ for some , we let the buyer switch to post-trading learning immediately and therefore the return policy can be designed to (1) influence the buyer's endogenous stopping; (2) provide transfer to the seller if the buyer requests a return.

Lemma 2. For a fixed v_{hb}, if the seller intends to induce stopping belief at, she has to provide selling mechanism $\{v_h\ \ v_{hb}; (x_r(\ ; v_{hb}); t_r(\ ; v_{hb}))\}$, where

$$x_r(\ ; v_{hb}) = \frac{V_1(\ ; v_{hb})}{v_h \quad v_l} \tag{3}$$

$$t_r(\ ; v_{hb}) = E(v|\ )\frac{V_1(\ ; v_{hb})}{v_h \quad v_l} \quad V(\ ; v_{hb}) \tag{4}$$

and $V_1(\ ; v_{hb})$ represents the partial derivative w.r.t .

Proof. Given Lemma 1, $V_B(\ ; v_{hb}) = V^0(\ ; v_{hb}) \quad V_P(\ ; v_{hb})$ on the support $[0; 1]$. For a fixed v_{hb}, we can pin down both $V_B(\ ; v_{hb})$ and $V^0(\ ; v_{hb})$ (the pink dotted curve in Figure 3). Therefore, to induce the buyer to stop at belief different from $\hat{\ }(v_{hb})$, $V_P(\ ; v_{hb})$ must be equal to $V^0(\ ; v_{hb})$. Otherwise, the buyer strictly prefers to stay in before-trading learning and does not stop. Furthermore, to ensure that it is incentive compatible for the buyer to stop at belief and request the return $(x_r; t_r)$, the buyer's expected payoff from requesting return, $E(v|\ )x_r \quad t_r$, should smoothly pass $V^0(\ ; v_{hb})$ at . Besides, the induced stopping belief must belong to the set $[\hat{\ }(v_{hb}); \bar{\ }(v_{hb})]$, in which $V^0(\ ; v_{hb}) = V(\ ; v_{hb})$. Above all, by the value matching condition: $E(v|\ )x_r \quad t_r = V(\ ; v_{hb})$, and the smooth pasting condition: $d[E(v|\ )x_r \quad t_r]/d \ = V_1(\ ; v_{hb})$, we obtain the expression of x_r and t_r. □

Figure 3: Inducing earlier stopping with Stochastic Return

Up to now, we can specify the return policy $(x_r; t_r)$, with each element as a function of v_{hb} and the stopping belief . Given this, the buyer optimally stops learning at belief and requests such return. For a xed selling price, the seller's expected revenue then becomes a function of the buyer's stopping belief and the seller eventually faces an information design problem. That is, if the seller prefers to encourage type-$_0$ buyer to learn, how much information the seller prefers the buyer to acquire, equivalently, at which belief the seller would like to guide the buyer to stop.

Note that for a xed $_0$, there exists a set of v_{hb} 2 $[\frac{k}{_0}; v_{hb}(_0)]$ that type- $_0$ buyer weakly prefers to learn. We call this *Learning-enforceable* constraint (LE). After choosing some v_{hb} subject to this constraint, the seller then has to decide at which belief he would like the buyer to stop learning and the stopping belief must belongs to $[\hat{}(v_{hb}); \bar{}(v_{hb})]$, which we called the *Stopping-inducible* constraint (SI). Meanwhile, we need the stopping belief $_0$ since the buyer's belief falls down if no signal arrives, and we simply call this the *Bayesian-plausible* condition (BP). Given Lemma 2, we can then write down the seller's revenue-maximization problem F().

$$\max_{\frac{k}{_0} \; v_{hb} \; v_{hb}(_0)} \; \max \quad (; v_{hb}) = \frac{_0}{1}(v_h \quad v_{hb}) + \frac{1 \quad _0}{1} t_r(; v_{hb}) \qquad (F)$$

$$\text{s.t.} \quad \hat{}(v_{hb}) \qquad \bar{}(v_{hb}) \qquad (SI)$$

$$_0 \qquad (BP)$$

Speci cally, with probability $\frac{1 \quad _0}{1}$, the buyer's posterior belief falls to and the seller obtains transfer $t_r(; v_{hb})$ from the buyer's request of return. With probability $\frac{_0}{1}$, a signal arrives before the buyer's posterior belief falls below , then the buyer consumes the item and the seller receives the selling price $v_h \quad v_{hb}$.

Before we investigate the solution, it is useful to mention the properties of the return transfer, $t_r(; v_{hb})$, which can be interpreted as the seller's safe revenue. Recall from Lemma 2, $t_r(; v_{hb}) = (\quad + \frac{v_l}{v_h \quad v_l})V_1(; v_{hb}) \quad V(; v_{hb})$.[19] It is easy to verify that (1) $t_r(; v_{hb})$ is increasing in the stopping belief , meaning that the seller can guarantee herself larger safe revenue by inducing a more optimistic stopping belief; (2) $t_r(; v_{hb})$ is increasing in v_{hb}, therefore by lowering price, the seller gets larger return transfer.

$$\frac{@ t_r(; v_{hb})}{@} = \frac{kE(vj \;)}{(1 \quad)^2(v_h \quad v_l)} > 0$$

$$\frac{@ t_r(; v_{hb})}{@ v_{hb}} = \frac{E(vj\hat{}(v_{hb}))}{(1 \quad \hat{}(v_{hb}))(v_h \quad v_l)} > 0$$

To see the intuition, for xed v_{hb}, to induce the buyer to stop learning at a more optimistic belief (larger), the seller has to increase the chance that the buyer keeps

[19] $t_r(; v_{hb}) = \frac{k v_l \quad v_l v_{hb} \quad k v_h \left[\log(\frac{}{1}) \quad \log\frac{k}{v_{hb} \quad k}\right]}{(v_h \quad v_l)}$.

the item upon requesting return,[20] and therefore the trading surplus of return is even larger. Though the buyer obtains larger surplus while requesting a return, it turns out the seller can also obtain larger return transfer. As for the monotonicity in v_{hb}, if the seller reduces the selling price and gives the buyer larger surplus if he observes a good news, then the buyer's continuation payo is shifted upwards. To induce the same stopping belief, the seller then need to o er the buyer larger chance of keeping the item while return. With similar reason, the seller is able to obtain larger return transfer given the total return surplus is increasing.

Denote $\bar{t}_r( ) := t_r(; \bar{v}_{hb}( ))$. It represents the largest return transfer that the seller could obtain by inducing the buyer to stop learning at belief . Given that $t_r(; v_{hb})$ is monotone in v_{hb}, the Learning-enforceable constraint $v_{hb} \in [\frac{k}{}; v_{hb}( )]$ has an equivalent graphical interpretation: for a xed , all return transfer belongs to the interval $[0; \bar{t}_r( )]$ is enforceable (see the dotted gray curves in Figure 4). It is interesting to notice that $t_r( )$ is increasing rst and then decreasing.[21] Hence, considering the graph of $t_r( )$ as an envelop, any pair of stopping belief and (non-negative) return transfer $(; t_r)$, that is weakly below this envelop, can be induced. In particular, there exists a unique v_{hb} and a unique x_r, with which the seller seeks return transfer t_r and the buyer optimally stops learning at . Furthermore, it is obvious that $t( ) > t_r( )$. Essentially, the selling price v_{hb} $v_{hb}( )$ must be higher than the corresponding return transfer $t_r(; \bar{v}_{hb}( ))$. Shown as in Figure 4, the red curve $t( )$ is always above the purple curve $t_r( )$.

Figure 4: Return transfers

[20] When $\in [\hat{}(v_{hb}); (v_{hb})]$, $V_1(; v_{hb}) > 0$ and $V_{11}(; v_{hb}) > 0$. Thus $x_r(; v_{hb})$ is increasing in .

[21] $\frac{d\bar{t}_r( )}{d} = \frac{@t(; \bar{v}_{hb}( ))}{@} + \frac{@t(; \bar{v}_{hb}( ))}{@\bar{v}_{hb}} \frac{d\bar{v}_{hb}( )}{d} = \frac{k}{(1 \quad) \quad (v_h \quad v_l)} \quad \frac{E(v| \quad)}{} \quad \frac{E(v|\tilde{}( ))}{1}$. The term in the bracket is decreasing. It's positive when $= \tilde{}( ) = \frac{L}{}$ and it's negative when $= \tilde{}( ) = \bar{}^{L}$.

5.1 Optimal learning encouragement for xed selling price

We rst focus on the a reduced problem, denoted as (?), which is the inner maximization of (F) with the Stopping-inducible constraint (SI).

$$\max_{\hat{\ }(v_{hb}) \quad \bar{\ }(v_{hb})} \quad (\ ; v_{hb}) \tag{?}$$

Its solution characterizes the optimal buyer's stopping belief that the seller is able to induce, given the selling price v_h v_{hb}. We denote $^{?}(v_{hb})$ as the global solution of problem (?), while (v_{hb}) denotes the local solution of the unconstrained objective function. Speci cally,

$$(v_{hb}) = f \quad : \frac{@(\ ; v_{hb})}{@} = 0 \text{ and } \frac{@^2 (\ ; v_{hb})}{@^2} \quad 0g:$$

Let $v_{hb}(\ )$ be the inverse function.[22] Denote $t_r(\ ) = t_r(\ ; v_{hb}(\ ))$, which represents the return transfer the seller obtains if he generates locally optimal stopping belief. $\Pr(\text{return}) = \frac{1 \quad 0}{1}$ is the ex-ante probability of return. Rearranging $\frac{@(\ ; v_{hb}(\ ))}{@} = 0$,[23] we can then obtain,

$$\underbrace{[v_h \quad v_{hb}(\ ) \quad t_r(\ ; v_{hb}(\ ))] \frac{d\Pr(\text{return})}{d}}_{\text{loss from more frequent refund}} = \underbrace{\Pr(\text{return}) \frac{@t_r(\ ; v_{hb}(\ ))}{@}}_{\text{gain from larger safe revenue}} \tag{FOC}$$

Note that v_h $v_{hb}(\ )$ $t_r(\ ; v_{hb}(\ ))$ is the amount of refund that the seller commits to make to the buyer who once requests a return, and $\frac{d\Pr(\text{return})}{d}$ is the marginal increment of return rate while the seller intends to enforce a larger stopping belief. Thus, the left-hand side is the marginal loss from reimbursing more often. For the right-hand side, $\frac{@t_r(\ ; v_{hb}(\ ))}{@}$ represents the marginal increment of return transfer, which can be interpreted as the marginal reduction of refund amount conditional on receiving a return request. Hence for to be the optimal stopping belief given $v_{hb}(\ )$, the seller has to balance the loss from more frequent return and the gain from smaller refund (equivalently, the gain from larger safe revenue). Besides, the FOC can be simpli ed as the following:

$$v_h \quad v_{hb}(\ ) \quad t_r(\ ; v_{hb}(\ )) = \frac{@t_r(\ ; v_{hb}(\ ))}{@}(1 \quad \ ) \tag{5}$$

This simpli cation can be directly perceived through Figure 4. Note the seller's expected revenue is a weighted average between the selling price v_h $v_{hb}(\ )$ and the transfer $t_r(\ )$. Graphically, the a ne curve crossing $(1; v_h \quad v_{hb}(\ ))$ and $t_r(\ )$ is tangent to $t_r(\ ; v_{hb}(\ ))$ at the point $(\ ; t_r(\ ))$. In particular, the dotted blue line is tangent to $t_r(\ ; v_{hb}^{000})$ as shown in Figure 4.

[22] It requires that (v_{hb}) to be monotone, which turns out to be true.

[23] The de nition of $v_{hb}(\ )$ implies that is the locally optimal solution of $(\ ; v_{hb}(\ ))$. Hence $\frac{@(\ ; v_{hb}(\ ))}{@} = [v_h \quad v_{hb}(\ ) \quad t_r(\ ; v_{hb}(\ ))] \frac{d\Pr(\text{return})}{d} + \Pr(\text{return}) \frac{@t_r(\ ; v_{hb}(\ ))}{@} = 0$.

Claim 1. The domain of $t_r(\ )$ is $[\_\ ; 0{:}5]$, where $\_ = \frac{k}{v\ h} + (\frac{k}{v\ h}(\frac{k}{v\ h} + \frac{v_l}{v_h\ \ v_l}))^{\frac{1}{2}} > \_^{L}$. $t_r(\ )$ is increasing in . $t_r(\_\ ) = 0$ and $t_r(0{:}5) = t_r(0{:}5)$.

Proof. The reader might be interested in the last statement $t_r(0{:}5) = t_r(0{:}5)$, so we prove it here. Otherwise, you can skip this proof. From the de nition of $v_{hb}(\ )$, the buyer is indi erent between consuming the item immediately and performing learning.

$$E(vj\ )\quad (v_h\quad v_{hb}(\ )) = V(\ ;\ v_{hb}(\ )) = E(vj\ )x_r(\ ;\ v_{hb}(\ ))\quad t_r(\ ;\ v_{hb}(\ ))$$

Which implies,[24]

$$v_h\quad v_{hb}(\ )\quad t_r(\ ;\ v_{hb}(\ )) = E(vj\ )[1\quad x_r(\ ;\ v_{hb}(\ ))] = \frac{kE(vj\ )}{(1\quad)\ (v_h\quad v_l)} \tag{6}$$

Recall that $\frac{@t(\ ;v_{hb})}{@} = \frac{kE(vj\ )}{(1\quad)\ ^2(v_h\quad v_l)}$ and substitute it into equation (5), we have

$$v_h\quad v_{hb}(\ )\quad t_r(\ ;v_{hb}(\ )) = \frac{kE(vj\ )}{^2(v_h\quad v_l)} \tag{7}$$

At the crossing point of $t_r(\ ;v_{hb}(\ ))$ and $t_r(\ ;\ v_{hb}(\ ))$, $v_{hb}(\ ) = v_{hb}(\ )$. Therefore, we can then solve $= 0{:}5$ as the unique solution that equation (6) and (7) equalize. The omitted proof are in the Appendix. □

Claim 1 describes the general properties of $t_r(\ )$, shown as the blue curve in Figure 4. The monotonicity of $t_r(\ ) = t_r(\ ;v_{hb}(\ ))$ is directly implied by (v_{hb}) being increasing.[25] To see the intuition, recall from the FOC, upon choosing the (locally) optimal stopping belief, the seller faces a trade-o between reimbursing less frequently (by inducing smaller stopping belief) and guaranteeing herself larger safe revenue (by inducing larger stopping belief). For a xed stopping belief, while increasing v_{hb}, the selling price becomes smaller but the return transfer is larger. In other words, the refund that the seller commits to pay is smaller. Hence the seller becomes less sensitive to the return frequency and his incentive to gain larger safe revenue becomes relatively substantial. Therefore, he optimally enforces a larger stopping belief (i.e., (v_{hb}) is increasing). However, the graph of $t_r(\ )$ pins down the region in which $t_r(\ )$ is an inducible return transfer (see Figure 4). In other words, $[\_\ ; 0{:}5]$ as the domain of $t_r(\ )$ characterizes the set where the local solution (v_{hb}) is also the global solution of problem (?).

Note that $t_r(\_\ ) = t_r(\_\ ; \frac{k}{\_})$ and $t_r(0{:}5) = t_r(0{:}5; v_{hb}(0{:}5))$. Therefore, when $v_{hb} > \bar{v}_{hb}(0{:}5)$ and $v_{hb} < \frac{k}{\_}$, the global solution of (?) is pinned down by the binding

[24] Where the second equality is directly obtained by substituting (ODE) and $E(vj\ )\quad (v_h\quad v_{hb}) = V(\ ;\ v_{hb})$ into equation (3).

[25] Take total derivative w.r.t , $\frac{dt_r(\ )}{d} = \frac{@t(\ ;v_{hb}(\ ))}{@} + \frac{@t(\ ;v_{hb}(\ ))}{@v_{hb}}\frac{dv_{hb}(\ )}{d} > \frac{@t(\ ;v_{hb}(\ ))}{@} > 0$. The monotonicity of $v_{hb}(\ )$ or (v_{hb}) implies the rst inequality, which then implies that the slope of $t_r(\ )$ is sleeper than $t_r(\ ;v_{hb})$. Hence $t_r(\ )$ crosses the horizontal axis at some $\_ > \_^{L}$.

Stopping-inducible constraint. More speci cally, when v_{hb} is su ciently high, $v_{hb} > \overline{v}_{hb}(0{:}5)$, the selling price is su ciently low, meanwhile the return transfer is su -ciently large. The refund $v_h \quad v_{hb} \quad t_r(\ ;v_{hb})$ then becomes su ciently small and to gain larger safe revenue becomes the seller's dominant incentive. Hence, she prefers to enforce stopping belief as large as possible, considering the stopping-inducible constraint, $^{-}(v_{hb})$ is then the (globally) optimal stopping belief. While $v_{hb} < \frac{k}{-}$, the refund becomes very large and then the dominant incentive is to reduce the return rate. However, the stopping-inducible constraint prevents the seller from inducing stopping belief smaller than $\hat{}(v_{hb})$. Therefore, $\hat{}(v_{hb})$ becomes the the optimal solution.[26]

Lemma 3. The solution of the objective function (?) is characterized as the following:

(1) When $v_{hb}\ 2\ [\underline{v}^L_{hb}; \frac{k}{-})$, $\ ^?\ 2\ f\ \hat{}(v_{hb}); ^{-}(v_{hb})g$.

(2) When $v_{hb}\ 2\ [\frac{k}{-}; \overline{v}_{hb}(0{:}5)]$, $\ ^?\ 2\ f\ \ (v_{hb}); ^{-}(v_{hb})g$.

(3) When $v_{hb}\ 2\ (\overline{v}_{hb}(0{:}5); \overline{v}^L_{hb}]$, $\ ^? = ^{-}(v_{hb})$.

While ignoring the Bayesian plausible constraint, this Lemma describes the optimal stopping belief that the seller would like to generate given the xed selling price. As a prelude of the optimal selling mechanism, we want to emphasize the intuition of these three potential solutions and the corresponding selling mechanism: (1) Stochastic Return with null learning, $^{-}(v_{hb})$; (2) Stochastic Return with partial learning, $\ (v_{hb})$; and (3) Free Return, $\hat{}(v_{hb})$. Note that here we view v_{hb} as some given number.

(1) Stochastic Return-Null Learning : Suppose $_0 < ^{-}(v_{hb})$, then inducing stopping at $^{-}(v_{hb})$ is not Bayesian plausible. While if $_0 > ^{-}(v_{hb})$, given the learning dynamic in Proposition 1, the buyer optimally consumes the item immediately. Hence $_0 = ^{-}(v_{hb})$ is the only possible case to induce stochastic return at $^{-}(v_{hb})$, meaning that type- $_0$ buyer exerts no learning. However, to induce stochastic return at $^{-}(v_{hb})$, the seller obtains revenue $t_r(^{-}(v_{hb}))$, which is smaller than $t(^{-}(v_{hb}))$, her revenue from Learning Deterrence. That is, Stochastic Return that induces null learning is never the optimal selling mechanism.

(2) Stochastic Return-Partial Learning : For xed selling price, the seller sets return policy $(x_r(\ (v_{hb}); v_{hb}); t_r(\ (v_{hb}); v_{hb}))$ to induce the buyer-optimal stopping at $\ (v_{hb}) > \hat{}(v_{hb})$. Essentially, the buyer optimally stops learning earlier and at a more optimistic belief than with the corresponding Free Return mechanism. Upon stopping at $\ (v_{hb})$, the buyer's continuation payo for keeping learning is still positive, therefore we call it partial learning. Note that by inducing a larger stopping belief and providing the buyer positive chance to keep the item when requesting return, the seller can also guarantee herself positive return transfer. Hence, though trading may not be

[26] With some extreme parameters, the right boundary point $^{-}(v_{hb})$ would be the global solution.

e cient ex-post, the seller can still obtain positive safe revenue.

(3) Free return-Full Learning : If the seller provides a Free Return mechanism, f v_h v_{hb}; (0; 0)g, the buyer has the option to opt-out and he optimally stops learning at ^(v_{hb}) when his continuation payo for keeping learning becomes zero. Hence, we call it full learning. In this case, the seller has to give away all selling price if the buyer indeed requests a return, meaning that there is no safe revenue. Because of this however, Free Return allows the seller to charge a higher price than Stochastic Return.

6 Optimal Selling Mechanism

For given prior belief, the seller can move v_{hb} to choose which selling scheme is of her best interest. For example, in Figure 5, when $_0$ = 0:5, the seller can choose a "No Return" contract with v_{hb} = $\overline{v}_{hb}(0{:}5)$ to deter learning and achieve e cient trading with certain revenue t(0:5) = v_h $\overline{v}_{hb}(0{:}5)$. While choosing "Stochastic Return" with smaller v_{hb} 2 ($\frac{k}{}$; $\overline{v}_{hb}(0{:}5)$), the seller can induce optimal level of partial learning, meanwhile guarantee positive revenue with probability one. Or she can use "Free Return" with even smaller v_{hb} $\frac{k}{}$, so that the buyer performs full learning and the seller obtains zero safe revenue. Hence, Learning Deterrence ensures the highest safe revenue but the lowest selling price, while Free Return ensures the highest selling price but the lowest safe revenue. Stochastic Return is in-between.

Figure 5: Feasible selling mechanisms

Theorem 1. Stochastic return is dominated by either Learning Deterrence or Free Return.

The formal proof is shown in the Appendix. Here we try to give an illustration of the main idea. Considering Figure 6, while incorporating the Bayesian plausible constraint, Stochastic Return (with locally optimal stopping belief) could be the potentially optimal selling mechanism when the buyer's prior belief $_0$ 2 [_ ; ($\frac{k}{}$)].[27] Recall from

[27] Suppose $_0$ < _ , then by LE condition, v_{hb} > $\frac{k}{}$. By Lemma 3, the locally optimal stopping

equation (5), the seller's expected revenue from the locally optimal Stochastic Return mechanism (; v_{hb}( )) consists of two parts: the safe revenue and her expected "bonus" from the buyer who once obtains a good signal.

$$(\ ; v_{hb}(\ )) = t_r(\ ; v_{hb}(\ )) + \frac{@t_r(\ ; v_{hb}(\ ))}{@ \ }(\ _0 \quad)$$

Note that with the locally optimal Stochastic Return mechanism, the induced stopping belief belongs to the domain of t_r( ), that is [_ ; 0:5]. Hence, nding the best Stochastic Return mechanism is equivalent with nding the best stopping belief (induced by a locally optimal Stochastic Return mechanism). Take derivative w.r.t. ,[28]

$$\frac{d(\ ; v_{hb}(\ ))}{d \ } = \frac{dt_r}{d \ } \quad \frac{@t_r}{@ \ } + (\ _0 \quad) \frac{@^2 t_r}{@ \ ^2} = [\quad (\ ) + \ _0 \quad] \frac{@^2 t_r}{@ \ ^2} \tag{8}$$

where ( ) = $\frac{(1 \quad)}{v_h}$E(vj$\frac{k}{v_{hb}(\ )}$). Within the support [_ ; 0:5], t_r is concave and therefore the monotonicity of (; v_{hb}( )) can be pin down by the sign of $_0$ ( ). In particular, if $_0$ > ( ), (; v_{hb}( )) is decreasing in , otherwise, it's increasing in .

Figure 6: (; v_{hb}( )) is quasi-convex

Claim 2. ( ) is supported on [_ ; 0:5], ^{0}( ) < 0, ^{00}( ) > 0 and ^{0}(0:5) > 1.

While in most cases, ^{0}(_) > 1,[29] and ( ) is positive, decreasing and convex, with slope larger than 1 in the whole support. Therefore, the a ne line $_0$ crosses ( ) for at most one time. In particular, when shifting up $_0$, we can nd two cuto s of $_0$: _ + (_) and 0:5 + (0:5). Such that,

belief should be larger than _ , violating the BP constraint. For $_0$ > ($\frac{k}{\_}$), the learning-enforceable v_{hb} < $\frac{k}{\_}$. By Lemma 3, the optimal stopping belief does not contain $^-$(v_{hb}).

[28] Note that @t_r=@ is independent of v_{hb}.

[29] In the Appendix, we consider the situation that ^{0}(_) < 1.

if $_0 <$ _ + (_), (; v $_{hb}$( )) is increasing in ;

if $_0$ 2 [_ + (_); 0:5 + (0:5)), (; v $_{hb}$( )) is quasi-convex in ;

if $_0$ 0:5 + (0:5), (; v $_{hb}$( )) is decreasing in .

This implies that the optimal Stochastic Return is at the boundary. More speci cally, for any prior belief $_0$ 2 [_ ; ($\frac{k}{}$)], the Stochastic Return mechanism that generates interior stopping belief , with _ < < minf 0:5; $_0$g, is dominated by either f v_h $\frac{k}{}$; (0; 0)g, a particular Free Return mechanism that induces stopping at _ , or Learning Deterrence.[30]

Intuitively, while choosing from the locally optimal stopping beliefs,[31] the seller faces a tension between guaranteeing herself larger safe revenue (by increasing the stopping belief) and increasing the expected "bonus" if a good news realized (by decreasing the stopping belief). Recall equation (8),

$$\frac{d(\; ; v_{hb}(\;))}{d} = \frac{dt_r}{d} \quad \frac{@t_r}{@} + (\; _0 \quad) \frac{@^2 t_r}{@^2} :$$

Note that $\frac{dt_r}{d}$, $\frac{@t_r}{@}$ and $\frac{@^2 t_r}{@^2}$ depend on but is independent of $_0$ and v_{hb}. That is, for a xed stopping belief , the marginal increment of safe revenue remains the same for all prior beliefs, however, the marginal loss from less expected "bonus" gets larger when the prior is larger. The later incentive becomes the dominant e ect after $_0$ rises above ( ) + ( ), meaning that the seller gains larger revenue by locally decreasing the stopping belief. Conversely, when $_0 <$ ( ), the seller prefers to locally increase the stopping belief. Note that ( ) is increasing in . Inversely, while xing the prior $_0$, = ^{1}($_0$) is then the critical point minimizing the seller's expected revenue (see the green dot in Figure 6). Hence, the seller prefers to locally decrease the stopping belief if < ^{1}($_0$), while he prefers to locally increase the stopping belief if > ^{1}($_0$) (see the green arrows in Figure 6). That is, the seller either prefers to induce smaller enough stopping belief (Free Return) or larger enough stopping belief (Learning Deterrence). Partial learning is never optimal.

Note that f v_h $\frac{k}{}$; (0; 0)g is just one particular Free Return mechanism. By changing the selling price, there are di erent Free Return mechanisms that induce di erent stopping beliefs. The reader might already notice that Free Return is a limit point of

[30] Considering $_0 <$ _ + (_), the best Stochastic Return is to induce stopping at $_0$ (incorporating BP) and the seller obtains revenue strictly less than with Learning Deterrence. If $_0$ 2 [_ + (_); 0:5], similar reasoning holds. For $_0$ 2 (0:5; 0:5 + (0:5)), suppose we ignore the LE constraint. Then the Stochastic Return that induces stopping at 0:5 generates expected revenue as a weighted average between t(0:5) and v_h v_{hb}(0:5) = t(0:5), which is smaller than t($_0$).

[31] Each is induced by a locally optimal Stochastic Return mechanism, i.e., stopping belief is locally optimal for given v_{hb}( ).

Stochastic Return for xed v_{hb}. By substituting $= \hat{} (v_{hb})$, $t_r(\hat{}(v_{hb}); v_{hb}) = 0$ and we can rewrite problem (F) to derive the best "Free Return" mechanism.[32] Denote the seller's revenue from "Free Return" as $^F(v_{hb})$.

$$\max_{v_{hb}} \quad {}^F(v_{hb}) \quad \frac{_0}{1} \frac{\hat{}(v_{hb})}{\hat{}(v_{hb})}(v_h \quad v_{hb}) \tag{F}$$

$$\text{s.t.} \quad \frac{k}{_0} \quad v_{hb} \quad v_{hb}(\ _0) \tag{LE}$$

We rst ignore the LE constraint. Denote $^F(\ _0) = \max_{v_{hb}} {}^F(v_{hb})$ and $v^F_{hb}(\ _0)$ as the optimal solution.[33] Explicitly,

$$v^F_{hb}(\ _0) = \frac{k}{} + \frac{\sqrt{k(\ _0 \quad 1)\ _0(k \quad v_h)}}{_0}$$

$$^F(\ _0) = \frac{2\sqrt{k(\ _0 \quad 1)\ _0(k \quad v_h)} + k \quad 2k\ _0 + \quad _0 v_h}{}$$

Easy to see that $^F(\ _0)$ is increasing on $_0$ and convex.[34] Denote I^F as the interval of prior beliefs that the seller's expected revenue from Learning Deterrence is lower than the best Free Return mechanism.

$$I^F = f\ _0 : t(\ _0) \quad {}^F(\ _0)g$$

Theorem 2. (Optimal selling mechanism): I^F is either an empty set or a closed interval, $I^F \quad (\underline{\ }^L; \overline{\ }^L)$. If $_0 2 I^F$, the Free Return mechanism $f v_h \quad v^F_{hb}(\ _0); (0; 0)g$ is optimal; if $_0 2 [\underline{\ }^L; \overline{\ }^L] n I^F$, Learning Deterrence $f\ \bar{\bar{t}}(\ _0); (1; \bar{\bar{t}}(\ _0))g$ is optimal.

For $f v_h \quad v^F_{hb}(\ _0); (0; 0)g$ to be an optimal selling mechanism for $_0 2 I^F$, the LE constraint, $\frac{k}{_0} \quad v_{hb} \quad v_{hb}(\ _0)$ from the constrained maximization (F) should be satis ed. The rst inequality can be reduced to $\hat{}(v^F_{hb}(\ _0)) \quad _0$, which holds when the seller's expected revenue is positive. Hence the rst inequality is automatically true when $^F(\ _0) \quad t(\ _0)$. Moreover, to check the second inequality $v^F_{hb}(\ _0) \quad v_{hb}(\ _0)$, when $^F(\ _0) \quad t(\ _0) = v_h \quad \overline{v}_{hb}(\ _0)$, we have $v_h \quad v^F_{hb}(\ _0) > {}^F(\ _0) \quad v_h \quad \overline{v}_{hb}(\ _0)$ which implies $v^F_{hb}(\ _0) < \overline{v}_{hb}(\ _0)$. Therefore, the maximum of $f\ \bar{\bar{t}}(\ _0); {}^F(\ _0)g$ pins down the optimal selling mechanism. In the Appendix, we show I^F is a closed interval if it's not empty and we denote it as $[\underline{\ }^F; \overline{\ }^F]$.

[32] Note that given $= \hat{}(v_{hb})$, the SI constraint is automatically true, meanwhile, the LE constraint implies $\quad _0$. Hence, we can also drop the BP constraint.

[33] One can verify that $^F(v_{hb})$ is concave, $\frac{d^2\ ^F}{dv^2_{hb}} = \frac{2k(\ _0 \quad 1)(k \quad v_h)}{(k \quad v_{hb})^3} < 0$. Besides, it is easy to verify that $v^F_{hb}(\ _0) \leq v_h$ can be implied by $\frac{k}{v_h} < \quad _0$. Hence the type-1 buyer's ex-post IR holds.

[34] $\frac{d^2\ ^F}{d\ ^2_0} = \frac{\sqrt{k = (\ 1 + \ _0)\ _0(k = \quad v_h)}}{2(1 \quad _0)^2\ ^2_0} > 0$

Figure 7: Optimal selling mechanism

This figure describes the seller's optimal choice of selling mechanism for all $\mu_0 \in [0;1]$ when I^F is not empty. For $\mu_0 < \underline{\mu}^L$ or $\mu_0 > \bar{\mu}^L$, the optimal selling mechanism is to set a non-refundable price equals the prior mean $\{E(v|\mu_0);(1;E(v|\mu_0))\}$, since an ex-ante sufficiently informative buyer optimally chooses to consume the item or quit the market for all selling mechanisms (see Proposition 2). For more uncertain prior belief, $\mu_0 \in [\underline{\mu}^L;\bar{\mu}^L]$, the seller optimally chooses between Learning Deterrence and Free Return. Essentially, she is comparing low price but larger probability of sale with high price but low probability of sale. To illustrate more, consider

$$\Delta^F t = \underbrace{Pr(sale|F)[(v_h - v_{hb}^F) - (v_h - v_{hb})]}_{\text{Gain from price difference}} - \underbrace{[1 - Pr(sale|F)](v_h - v_{hb})}_{\text{Loss from return}}$$

where $Pr(sale|F)$ is the probability of successful sale conditional on the optimal Free Return $\{v_h - v_{hb}^F(\mu_0);(0;0)\}$. $Pr(sale|F)$ is increasing in the prior belief.[35] Switching from Learning Deterrence to Free Return, the seller gives away the probability of sale to increase the selling price. When the buyer's prior is sufficiently optimistic (i.e., $\mu_0 > \bar{\mu}^F$), even with Learning Deterrence, the seller is able to charge the buyer a relatively high price and the price difference between Learning deterrence and Free Return is not that important. Therefore it is not optimal for seller to sacrifice trading opportunity just to increase the price a little bit. With sufficiently pessimistic prior (i.e., $\mu_0 < \underline{\mu}^F$), the seller can expect a small chance of successful sale and hence a large chance of ex-post return. Therefore, the seller is willing to sacrifice the price to assure efficient trading.

Figure 8 provides an overview of the optimal selling mechanism when k changes (refer to different columns). In particular, the black curve represents "No Return" with price equals prior mean $\{E(v|\mu_0);(1;E(v|\mu_0))\}$, the green curve represents Learning Deterrence $\{\bar{\bar{t}}(\mu_0);(1;\bar{\bar{t}}(\mu_0))\}$, while the red curve represents Free Return $\{v_h - v_{hb}^F(\mu_0);(0;0)\}$. For each column from top to bottom, we depict the seller's expected revenue given different selling mechanisms, the selling price in the optimal selling mechanism, and the buyer's ex-ante trading surplus given the optimal selling mechanism.

[35] $Pr(sale|F) = \frac{\mu_0 \hat{\Phi}(v_{hb}^F(\mu_0))}{1 - \hat{\Phi}(v_{hb}^F(\mu_0))}$. $\frac{d\,Pr(sale|F)}{d\mu_0} = 1 - \sqrt{\frac{k}{v_h - k}}\frac{1}{2}\frac{1 - 2\mu_0}{\sqrt{(1-\mu_0)\mu_0}} > 0$ since $\mu_0 > \frac{k}{v_h}$.

Figure 8: Comparative statics

Consider the first raw of Figure 8, we picture the seller's expected revenue from Learning Deterrence (green curve) and Free Return (red curve) on the domain $[\underline{\ }^L; \bar{\ }^L]$ for different $\frac{k}{}$. We can interpret $\frac{k}{}$ as a measure of learning efficiency. Essentially, larger (smaller) $\frac{k}{}$ indicates less (more) efficient learning (e.g., lower learning cost or higher rate of signal occurrence). Comparing the three graphs in the first raw, when learning becomes more efficient, the seller's revenue from Leaning Deterrence (green curve) falls down but her revenue from Free Return (red curve) shifts up. The two curves will intersect at some point and the interval I^F expends when $\frac{k}{}$ becomes smaller (see Proposition 6 in the Appendix). That is, the seller optimally offers Free Return with a larger set of prior beliefs when learning is more efficient. Intuitively, with more efficient learning, the buyer is more tempting to learn. To deter buyer's private learning, the seller has to provide more surplus to the buyer and then obtains less revenue. However, with Free Return, the buyer is willing to perform longer learning and there will be larger chance of ex-post successful sale, which implies larger revenue for the seller. As a result, Free Return dominates Learning Deterrence in a larger set of prior belief when learning becomes more efficient. Furthermore, the seller's expected revenue is increasing in the buyer's prior belief regardless of the learning efficiency (see the maximum of the two curves in the first raw).

As shown in the second raw, the selling price is non-monotone when I^F is not empty.

In particular, at the end points of I^F, the selling price has a discontinuous jump, since the seller's revenue is the same from both Learning Deterrence and Free Return, but she receives return request with strictly positive probability. The discontinuous jump in the selling price turns out to imply discontinuous reduction in the buyer's ex-ante trading surplus (see the third raw), since the buyer obtains less trading surplus even if he is realized to have high matching value, which implies less continuation payoff. To illustrate more, when $\mu_0 \in [\underline{\mu}^F; \bar{\mu}^F]$ and the seller optimally chooses Free Return, the buyer then incurs strictly positive learning cost and trading is inefficient. Therefore, the total trading surplus is smaller than the prior mean (Learning Deterrence ensures that total trading surplus equals the prior mean). Meanwhile, the seller obtains larger revenue with Free Return than with Learning Deterrence. Therefore, the buyer's expected trading surplus is shifted down with Free Return.

Proposition 5. Let $\pi^o(\mu_0)$ be the seller's expected revenue from the optimal selling mechanism.

$$\lim_{k \to 0} \pi^o(\mu_0) = \begin{cases} v_l; & \mu_0 < \frac{v_l}{v_h} \\ \mu_0 v_h; & \mu_0 \geq \frac{v_l}{v_h} \end{cases}$$

Consider the standard result of selling a single indivisible good, where the buyer has private valuation $v \in \{v_h; v_l\}$ and the seller attaches prior distribution μ_0 as the probability of high valuation buyer. The seller values the item with zero utility. We can find the revenue maximization mechanism: (mass market) when $\mu_0 < \frac{v_l}{v_h}$, setting price equals v_l; (niche market) when $\mu_0 \geq \frac{v_l}{v_h}$, setting price equals v_h. Intuitively, when the seller believes that there is a larger chance to match with a high value buyer, he is willing to charge a high price and sacrifice the trading opportunity with the low value buyer, vise versa. In our setup, when k goes to zero, the seller's revenue converges to the revenue he can gets with a privately perfect-informed buyer. This is because, if k converges to zero, then the buyer can learn almost perfect information, therefore with Free Return, the seller can set the selling price arbitrarily close to v_h and let go the buyer almost sure to have low valuation. While with Leaning Deterrence, the seller has to set the price arbitrarily close to v_l, otherwise, the buyer always have incentive to learn to avoid consuming the item when his true valuation is low. The ratio $\frac{v_l}{v_h}$ pins down the cutoff of prior belief that the seller is indifferent between Free Return and Learning Deterrence.

7 Conclusion

This paper discusses the seller's optimal pricing and return policy when the buyer can privately acquire information to resolve his uncertainty on a certain product. The pricing and return policy is designed to induce the optimal level of buyer's private learning. We find that the optimal selling mechanism is either to allow free return that induces full learning or to deter the buyer's private learning. Conditional on optimal selling mechanism, the buyer's trading surplus turns out to be non-monotone in his prior belief. However, our results rely on the assumption that buyer's prior belief is common information. Therefore, it naturally brings out the next question: what if the buyer has private information *a priori*? Then clearly he has incentive to misreport his private information. Furthermore, if the seller can design a time-contingent return policy, is it possible for her to use it to screen the buyers endowed with different prior beliefs? We believe all these are interesting extensions.

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Appendix

Lemma 1

Proof. First note that a return policy that intends to generate continuation payo such that

$$
\begin{aligned}
V_P(\ (\ )) = & \quad kd \ + \ (\ ) \, d \ v_{hb} + (1 \quad (\ ) \, d \ ) V_P(\ (\ + d \ )) \\
< & \quad kd \ + \ (\ ) \, d \ v_{hb} + (1 \quad (\ ) \, d \ ) V_B(\ (\ + d \ )) = V_B(\ (\ ))
\end{aligned}
$$

cannot be realized. Otherwise, type-( ) buyer strictly prefers before-trading learning. Therefore, it is without loss to focus on return policies that the above inequality binds. In particular, $V_P(\ ) = V(\ )$ implies $V_B(\ ) = V_P(\ )$. We are going to show that it is not optimal for the seller to induce

$$V_B(\ ) = V_P(\ ) = V(\ ) \quad V^0(\ )$$

$$\text{s.t. strict inequality holds for some :}$$

Prove by contradiction. First, conditional on this, the buyer will switch to post-trading learning at some time and the switch time does not a ect the seller's expected revenue.[36] To see this, from the ex-ante point of view, regardless of switching time, the buyer stops learning till either of the following two events: a) a signal arrives and the buyer purchases the item at price t_b without requesting a return; b) no signal arrives and the buyer stops at the same stopping belief as in the post-trading learning, requesting some return $(x_r; t_r) \neq (1; t_b)$. Therefore, the ex-ante probabilities of event a)-receiving price t_b and b)-receiving transfer t_r are the same regardless of the switching time and we can assume all learning are performed after trading.

Second, given the standard result of optimal stopping time with exponential bandit, the smooth pasting and value matching condition should be satis ed upon optimal stopping at .

$$V_P^0(\ ) = (v_h \quad v_l) x_r \qquad \text{(smooth pasting)}$$

$$V_P(\ ) = E(vj \ ) x_r \quad t_r \qquad \text{(value matching)}$$

Hence, the transfer that the seller can obtain at the buyer's optimal stopping equals the allocation value minus the buyer's surplus $t_r = E(vj \ ) \frac{V_P^0(\ )}{v_h \quad v_l} \quad V_P(\ )$. Recall that conditional on learning, the ODE holds regardless of the return policy. Hence for a xed , if $V_P(\ )$ is decreasing, $V_P^0(\ )$ is increasing. Hence the seller can obtains larger transfer t_r by reducing the buyer's surplus $V_P(\ )$. Contradiction of optimality. Therefore it is optimal to reduce $V_P(\ )$ till it equals $V^0(\ )$. □

[36] It is not optimal to stop before-trading learning by quitting, otherwise $V_B(\ ) = V^0(\ )$ for all .

Proposition 1

Proof. (1) is from the learning feasibility constraint (LF). (2) and (3) are implied by the following Claim.

Claim 3. Let ~( ) := $\frac{k}{\bar{v}_{hb}(\ )}$ with support [_ᴸ; ⁻ᴸ]. ~( ) for the whole support and the equality holds only at the two ending points. ~( ) is increasing and symmetric about the line 1- . ~( ) is increasing and then decreasing.

Recall that $^{-}(v_{hb})$ = f : V(; v $_{hb}$) = v $_h$ + (1)v_l (v_h v_{hb})g and $\bar{v}_{hb}(\ )$ = $^{-\ 1}(\ )$. Hence, by setting type-1 buyer's surplus to be $\bar{v}_{hb}(\ )$, a type- buyer is indi erent between accepting the price or exerting learning till observing a signal or quitting and ~( ) is the optimal quitting belief. In other words, if the buyer's prior is , then there does not exists a selling mechanism t_b; (1; t_b)g can achieve learning belief smaller than ~( ). From V(; $\bar{v}_{hb}(\ )$) = E(vj) (v_h $\bar{v}_{hb}(\ )$), we can have,

$$\frac{d\bar{v}_{hb}}{d} = \frac{k(k \quad \bar{v}_{hb}(\ ))}{^2(1 \quad)^2\ \bar{v}_{hb}(\ )} = \frac{k[\sim \quad 1]}{(1 \quad)^2} < 0$$

Therefore,

$$\frac{d\sim}{d} = \frac{dk= \bar{v}_{hb}(\ )}{d} = \frac{k^2(\ \bar{v}_{hb}(\ ) \quad k)}{^3(1 \quad)^2\ \bar{v}_{hb}(\ )^3} = \frac{\sim^2(1 \quad \sim)}{(1 \quad)^2}$$

which is a di erential equation. And we can pin down the integral by substituting the boundary point (_ᴸ; _ᴸ) into the general solution.[37]

$$\frac{1}{\sim} \quad \log[1 \quad \sim] + \log[\sim] = \frac{1}{1} \quad \log[1 \quad] + \log[\ ] \quad \frac{(v_h \quad v_l)}{k} \qquad (9)$$

Denote the LHS as f(~) and the RHS as g( ). The domain of both functions is [_ᴸ; ⁻ᴸ] and f( ) = g( ) at the two end points. Note that f ^{0}( ) > g ^{0}( ) when the both arguments are smaller then 0.5 and f ^{0}( ) < g ^{0}( ) when both arguments are larger then 0.5.[38] Therefore f( ) and g( ) cross only at those two boundary points and therefore ~( ) < for all 2 (_ᴸ; ⁻ᴸ). For ~( ) to be symmetric about 1 , note that the re ection point of (; ~) over line 1 is (1 ~; 1). And it is easy to check that for any point (; ~) that equation (9) holds, by substituting (1 ~; 1), equation (9) still holds.

Note that ~^{0}(_ᴸ) < 1 and ~^{0}(⁻ᴸ) > 1, if ~^{0}( ) 1 is single crossing, then ~( ) is increasing and then decreasing. To show this, note that $\frac{d\sim}{d}$ = $\frac{\sim^2(1 \quad \sim)}{(1 \quad)^2}$ = 1 implies ~^{2}(1 ~) = (1)^2 . And the graph of ~^{2}(1 ~) (blue curve) is a re ection of the graph of (1)^2 (orange curve) over line = 0:5.

[37] The general solution is $\frac{1}{\sim}$ log[1 ~] + log[~] = $\frac{1}{1}$ log[1] + log[ ] + C. Though the integral crosses both (_ᴸ; _ᴸ) and (⁻ᴸ; ⁻ᴸ), _ᴸ = 1 ⁻ᴸ and therefore either of these two boundary points can pin down the integral.

[38] f ^{0} = $\frac{1}{\sim^2}$ + $\frac{1}{\sim(\sim \quad 1)}$ and g ^{0} = $\frac{1}{(1 \quad)^2}$ + $\frac{1}{(\quad 1)}$.

Since $\tilde{}(\ ) <\ $, therefore to have $[\tilde{}(\ )]^2(1\ \ \tilde{}(\ )) = (1\ \ \ )^2$, we must have $\tilde{}(\ ) = 1$, meaning that the solution of $\tilde{}^0(\ ) = 1$ is on the line 1 . Given that $\tilde{}(\ )$ is symmetric about 1 which crosses $\tilde{}(\ )$ only once, $\tilde{}^0(\ ) = 1$ has a unique solution. □

Proposition 4

For the rst part, recall that $V(\ ;\ v_{hb}(\ )) = v_{hb}(\ )\ \ (1\ \ \ )(v_h\ \ v_l)$. Taking derivative w.r.t , we have

$$\frac{dV(\ ;\ v_{hb}(\ ))}{d} = (v_h\ \ v_l)\quad A\frac{(1\ \ \tilde{})}{(1\ \ \ )^2} + 1$$

where $A = \frac{k}{(v_h\ \ v_l)} = (1\ \ \underline{\ }^{L})\underline{\ }^{L}$. Hence it is easy to see $\frac{dV(\ ;\ v_{hb}(\ ))}{d} = 0$ at $\underline{\ }^{L}$ or $\overline{\ }^{L}$. Since $V(\ ;\ v_{hb}(\ )) > 0$ when $2\ (\underline{\ }^{L};\overline{\ }^{L})$, to show $V(\ ;\ v_{hb}(\ ))$ is increasing rst and then decreasing, we only need $V^0(\ ;\ v_{hb}(\ ))$ crosses 0 only once for $2\ (\underline{\ }^{L};\overline{\ }^{L})$. Denote $(\ ) = \frac{A\ \ (1\ \ \ )^2}{A}$.[39] We have $(\ ) = \tilde{}(\ )$ at $\underline{\ }^{L}$ or $\overline{\ }^{L}$ and we want to show $(\ )$ crosses $\tilde{}(\ )$ only once for $2\ (\underline{\ }^{L};\overline{\ }^{L})$.

From the proof of Claim 3, $\tilde{}(\ )$ is the solution of the below equation:

$$\underbrace{\frac{1}{\tilde{}}\ \log[1\ \ \tilde{}] + \log[\tilde{}]}_{f(\tilde{})} = \underbrace{\frac{1}{1}\ \log[1\ \ \ ] + \log[\ ]\ \ \frac{(v_h\ \ v_l)}{k}}_{g(\ )}$$

Replacing $\tilde{}$ by $(\ )$, we get $(f\ \ \ )(\ )\ \ g(\ ) = 0$, which is clearly true at $\underline{\ }^{L}$ or $\overline{\ }^{L}$. Taking derivative w.r.t , we have

$$(f\ \ \ )^0(\ )\ \ g^0(\ ) = \frac{1}{(1\ \ \ )^2}\ \frac{A^2(3\ \ \ 1)(1\ \ \ )}{[A\ \ (1\ \ \ )^2\ ]^2}\ \ 1$$

Denote $y(\ ) = A^2(3\ \ \ 1)(1\ \ \ )$ and $z(\ ) = [A\ \ (1\ \ \ )^2\ ]^2$. If $y(\ ) > z(\ )$, $(f\ \ \ )(\ )\ \ g(\ )$ is increasing, otherwise it's decreasing. Note that $y(\underline{\ }^{L}) < z(\underline{\ }^{L})$ and $y(\overline{\ }^{L}) < z(\overline{\ }^{L})$.[40] $y(\ )$ is a second order polynomial function that is negative when $< 1=3$, and it increases on if $< 2=3$ and decreases on if $> 2=3$. $z(\ )$ is a high order polynomial function whose rst order derivative equals zero at $1=3;1$ and

[39] From $A\frac{(1\ \ \ )}{(1\ \ \ )^2} + 1 = 0$.

[40] Easy to check by substituting $A = (1\ \ \underline{\ }^{L})\underline{\ }^{L}$ or $A = (1\ \ \overline{\ }^{L})\overline{\ }^{L}$.

the roots of $(1\ \ )^2\ \ A = 0$ (at most two roots).[41] We can show that $z(\ )$ double crossing $y(\ )$ in the support $[\_^{L}; {}^{-L}]$, and while increasing , $z(\ ) > y(\ )$ first and then $z(\ ) < y(\ )$ and then $z(\ ) > y(\ )$.[42] Therefore, $(f\ \ )(\ )\ \ g(\ )$ decreases first and then increases and then decreasing, given $(f\ \ )(\ )\ \ g(\ ) = 0$ at the two boundary points, $(f\ \ )(\ )\ \ g(\ ) = 0$ has a unique interior solution, which implies $(\ )$ crosses $\tilde{}(\ )$ only once for $\ 2\ (\_^{L}; {}^{-L})$.

The properties of $t(\ _0)$ can be directly obtained from the properties of $V(\ _0; v_{hb}(\ _0))$, since $t(\ _0) = E(v|\ _0)\ \ V(\ _0; v_{hb}(\ _0))$.

Claim 1

To show the second part, for $\frac{@(\ ;v_{hb})}{@} = 0$, by implicit theorem, we have

$$\frac{dv_{hb}(\ )}{d} = \frac{k[(2\ \ 1)\,v_h\ \ 2(1\ \ )^2 v_l](k\ \ v_{hb}(\ ))}{{}^2(1\ \ )\ {}^3 v_h v_{hb}(\ )}$$

Note that

$$\frac{d\bar{t}_r(\ )}{d} = \frac{@t_r(\ ;\ \bar{v}_{hb}(\ ))}{@} + \frac{@t_r(\ ;\ \bar{v}_{hb}(\ ))}{@\bar{v}_{hb}}\frac{d\bar{v}_{hb}(\ )}{d}$$

$$\frac{dt_r(\ )}{d} = \frac{@t_r(\ ;v_{hb}(\ ))}{@} + \frac{@t_r(\ ;v_{hb}(\ ))}{@v_{hb}}\frac{dv_{hb}(\ )}{d}$$

Notice that $\frac{@t_r(\ ;v_{hb})}{@}$ only depends on and when $\ \ 0{:}5$, $\frac{dv_{hb}(\ )}{d} > 0$ and $\frac{dv_{hb}(\ )}{d} < 0$. Therefore $\frac{d\bar{t}_r(\ )}{d} < \frac{dt_r(\ )}{d}$ for $\ \ 0{:}5$. Therefore, while reducing , $t_r(\ )$ decreases faster than $\bar{t}_r(\ )$. Denote $\_$ as the solution that $t_r(\ ) = 0$. While increasing from 0.5, $t_r(\ )$ is above $\bar{t}_r(\ )$ because of single-crossing. From the SI constraint, the graph of $t_r(\ )$ is below $\bar{t}_r(\ )$, hence $t_r(\ )$ is supported on $[\_\ ; 0{:}5]$ and $t_r(\ )$ is increasing on this support.

To pin down $\_$, notice that $\frac{@(\ ;v_{hb}(\ ))}{@}|_{\_} = 0$ and $\_\ = \frac{k}{v_{hb}(\_)}$. By plugging $v_{hb} = \frac{k}{}$

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[41] $z^0(\ ) = 2[(1\ \ )^2\ \ A](3\ \ 1)(\ \ 1)$. The derivative of $(1\ \ )^2\ \ A$ is $(3\ \ 1)(\ \ 1)$. Hence $(1\ \ )^2\ \ A$ is increasing if $\ < 1{=}3$ and decreasing afterwards. $(1\ \ )^2\ \ A < 0$ when $= \_^{L}; {}^{-L}$. The maximum of $(1\ \ )^2$ is $4{=}27$, which is not necessarily larger than A.

[42] (1) Suppose there are two distinct roots for $(1\ \ )^2\ \ A$, denoted as $r_1; r_2$, then $r_1 < 1{=}3$, $r_2 > 1{=}3$. Hence, $z(\ ) > y(\ )$ for $\ < 1{=}3$, $z(1{=}3) > 0 > y(1{=}3)$, $z(r_2) = 0 < y(1{=}3)$ and $z({}^{-L}) > y({}^{-L})$. Note that $z^{00}(\ ) = 2[(3\ \ 1)(1\ \ )]^2 + 2((1\ \ )^2\ \ A)(6\ \ 4)$, and $y^{00} < 0$, $y^0(\ ) < 0$ when $\ > 2{=}3$. If $z(\ )$ crosses $y(\ )$ between r_2 and $2{=}3$, meaning $r_2 < 2{=}3$, then at the crossing point $z^{00}(\ ) > 0$ and they won't cross again. If they cross after $2{=}3$, then $y(\ )$ is decreasing and $z(\ )$ is increasing after $2{=}3$ and they won't cross again. Therefore $z(\ )$ double crosses $y(\ )$. (2) Suppose there is a unique root, then $r_1 = r_2 = 1{=}3$. Hence $z(\ ) > y(\ )$ for $\ < 1{=}3$, $z(1{=}3) = y(1{=}3)$, $z^0(1{=}3) = 0 < y^0(1{=}3)$ and $z({}^{-L}) > y({}^{-L})$. Also $z^{00}(\ ) > 0$ on $[1{=}3; 2{=}3]$. Therefore $z(\ )$ double crosses $y(\ )$. (3) Suppose there is no root, then $A\ 2\ (4{=}27; 1{=}4)$, $z^0(\ ) < 0$ when $\ < 1{=}3$ and $z^0(\ )\ \ 0$ when $\ \ 1{=}3$. We can check that $z(1{=}2) < y(1{=}2)$ for $A\ 2\ (4{=}27; 1{=}4)$, hence given $y(\_^{L}) < z(\_^{L})$ and $y({}^{-L}) < z({}^{-L})$, we have the same double crossing.

into $\frac{\partial(\ ;v_{hb})}{\partial}$, we have

$$\frac{\partial(\ ;v_{hb})}{\partial}|_{v_{hb}=\frac{k}{}} = \frac{(\ _0\ \ 1)(\ ^2 v_h(v_h\ \ v_l)\ \ k(2\ (v_h\ \ v_l)+v_l))}{(1\ \ )^2\ ^2(v_h\ \ v_l)}$$

Since $^2 v_h(v_h\ \ v_l)\ \ k(2\ (v_h\ \ v_l)+v_l)$ is increasing on $>\ 0$,[43] and it is negative when is small and is positive when is large. Hence $\frac{\partial(\ ;v_{hb})}{\partial}|_{v_{hb}=\frac{k}{}}$ is positive when is small and negative when is large. Let $\frac{\partial(\ ;v_{hb})}{\partial}|_{v_{hb}=\frac{k}{}} = 0$, we can pin down $\_ = \ + \sqrt{\ (\ +\frac{v_l}{v_h\ v_l})}$.

Now we verify that $\frac{\partial^2(\ ;v_{hb})}{\partial^2}\ \ 0$ holds on the support $[\_ ; 0{:}5]$.

$$\frac{\partial(\ ;v_{hb})}{\partial} = \frac{(1\ \ _0)}{(1\ \ )^2(v_h\ \ v_l)}\left[v_h(\ v_h+v_{hb}+v_l)+\frac{k(\ (v_h\ \ 2v_l)+v_l)}{2}+\frac{kv_h(\log[\frac{}{1}]\ \ \log[\frac{k}{v_{hb}\ \ k}])}{}\right]$$

Denote the term in the bracket as $(\ )$. $\frac{d}{d} = \frac{k(1\ 2\ )v_h+2k(1\ \ )^2 v_l}{(\ \ 1)^3}$. $\frac{\partial(\ ;v_{hb})}{\partial}|_{v_{hb}=\frac{k}{}}$ is positive when $<\ \_$ and $\frac{d}{d}$ is negative when $\ \ 0{:}5$, and since we have already pin down $t_r(\ )$ which is located to the left of $=0{:}5$, therefore the SOC holds on this support $[\_ ; 0{:}5]$. The reader might also be interested in the case that $>\ 0{:}5$. Note that on the domain $\ \in (0;1)$, the numerator of $\frac{d}{d}$ is negative when is small and could be positive when is large. Hence $(\ ;v_{hb})$ for fixed v_{hb} is one of the following: (1) being increasing and then (weakly) decreasing on;[44] (2) being increasing and decreasing and then increasing on; (3) keeping decreasing; and (4) being decreasing first and then increasing. In the first two cases, $t_r(\ )$ pins down the local optimization.

Claim 2

Denote $E(\ ) := E[v|\frac{k}{v_{hb}(\ )}]$. First, we want to show, for $\ \in [\_ ; 0{:}5]$, $\frac{d^2 v_{hb}(\ )}{d^2} <\ 0$, $E'(\ ) < 0$ and $E''(\ ) > 0$. Note that $\frac{d^2 v_{hb}(\ )}{d^2}$ equals to

$$\frac{k(k\ \ v_{hb})}{^2(1\ \ )^2\ ^6 v_h^2 v_{hb}}\left[(\frac{k}{v_{hb}})^2(\ (v_h\ \ 4v_l)\ \ 2\ ^2(v_h\ \ v_l)+2\ v_l)^2+\ ^2\ \ v_h[(\ 2+(5\ \ 4\ )\ )v_h+\frac{2(1\ \ )^2(\ 3+2\ )v_l}{}\right]$$

[43] The derivative of $^2 v_h(v_h\ \ v_l)\ \ k(2\ (v_h\ \ v_l)+v_l)$ is $2(k\ \ v_h\ )(v_h\ \ v_l) > 0$.

[44] Weakly decreasing if $=0{:}5$ is the boundary point.

To simplify exposition, denote B ($(v_h$ $4v_l)$ 2 $^2(v_h$ $v_l) + 2v_l)^2$ and C $v_h[($ $2 + (5$ 4) $)v_h + \frac{2(1 \quad)^2(\quad 3+2 \quad)v_l}{}]$. To show $\frac{d^2 v_{hb}(\ )}{d \quad ^2} < 0$, we need

$$(\frac{k}{v_{hb}})^2 B + \ ^2 C < 0$$

Note that B is positive and C is negative.[45] Moreover, $(\frac{k}{v_{hb}})^2 <$ ^{2}, and we can show $B + C < 0$,[46] therefore $v_{hb}(\ )$ is concave. For $E^0(\ ) < 0$, since $E(\ )$ is the expected value of type $\frac{k}{v_{hb}(\ )}$ buyer. Since $v_{hb}(\ )$ is increasing on and therefore $\frac{k}{v_{hb}(\ )}$ is decreasing on , so does $E(\ )$. Notice $E^{00}(\ ) = \frac{k(v_h \quad v_l)}{} \frac{(v_{hb})^{00}(v_{hb})^2 \quad 2[(v_{hb})^0]^2 v_{hb}}{(v_{hb})^4} > 0$

$$
\begin{aligned}
{}^0(\ ) &= \frac{1}{v_h}[E(\ ) \quad (1 \quad)E^0(\ )] \\
&= \frac{1}{v_h} \Big" E(\_ \ ) + \int_{\_} E^0(\ )d \quad (1 \quad)E^0(\ ) \Big\#
\end{aligned}
$$

Since $E^0(\ )$ is negative and increasing on , $\int_{\_} E^0(\ )d$ $(1$ $)E^0(\ )$ is decreasing on and therefore $^0(\ )$ is increasing on , equivalently, $^{00}(\ ) > 0$.

Denote ^ ( ) := $\hat{}(v_{hb}(\ ))$. To show $^0(\frac{1}{2}) >$ 1, simplify $^0(\frac{1}{2})$, we have:

$$^0(\frac{1}{2}) = \quad \frac{4(v_h \quad v_l)v_l \hat{} \ (\frac{1}{2})^2}{v_h^2} \quad 1 \quad \hat{} \ (\frac{1}{2}) \quad + \frac{1}{v_h}E[v j \hat{} \ (\frac{1}{2})]$$

We can show $^0(\frac{1}{2})$ is decreasing in v_l and hence plugging $v_l = 0$ and $v_l = v_h$ into $^0(\frac{1}{2})$, we have $^0(\frac{1}{2})j_{v_l=0} = \ \hat{} \ (\frac{1}{2}) >$ 1 and $^0(\frac{1}{2})j_{v_l \, ! \, v_h} =$ 1.[47]

Theorem 1

We prove this theorem by discussing $_0$ case by case. In particular, we divide $[\_^L; \bar{}^L]$ into three partitions, $[\_^L; \_ \ ) \ [\ [\_ \ ; \bar{}(\frac{k}{\_})) \ [\ [\bar{}(\frac{k}{\_}); \bar{}^L]$.

Case 1: When $_0 \, 2 \, [\_^L; \_ \ )$, for fixed v_{hb} such that $\frac{k}{\_} < \frac{k}{_0}$ v_{hb} $v_{hb}(\ _0)$, $(\ ; v_{hb})$ is increasing on for $< \ _0$. Therefore it is optimal to let the buyer stop learning at his prior belief and there is no successful sale and the seller obtains revenue $t_r(\ _0; v_{hb})$. Since $t_r(\ _0; v_{hb})$ is increasing on v_{hb}. Hence the highest revenue is $t_r(\ _0) < t(\ _0)$. Therefore, when $_0 \, 2 \, [\_^L; \_ \ )$, the optimal selling mechanism is Learning Deterrence.

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[45] ($2 + (5$ 4)) reaches its maximum $0{:}5$ when $= 0{:}5$ and $\frac{2(1 \quad)^2(\quad 3+2 \quad)v_l}{}$ is negative.

[46] To show this, $B + C =$ $2(1$ $)[$ $(1 + 2($ $1)$ $)v_h^2 + (1$ $)(3 + 4($ $1)$ $)v_h v_l + 2($ $1)^3 v_l^2]$. It is easy to verify that $(1 + 2($ $1)$ $)v_h^2 > 0$ and $(3 + 4($ $1)$ $)v_h v_l + 2($ $1)^3 v_l^2 > (3 + 4($ $1)$ $)v_h v_l + 2($ $1)^3 v_h v_l = v_h v_l(1 + 2$ $(1 + ($ $1)$ $)) > 0$.

[47] $\frac{d \ ^0(\frac{1}{2})}{dv_l} = \frac{1}{v_h^2}[(\hat{} \ (\frac{1}{2}) \quad 1)(v_h + 4(v_h \quad 2v_l)\hat{} \ (\frac{1}{2})^2) \quad (v_h \quad v_l)(v_h + 4v_l(2 \quad 3\hat{} \ (\frac{1}{2}))\hat{} \ (\frac{1}{2}))\frac{d\hat{} \ (\frac{1}{2})}{dv_l}] < 0$ since $\hat{} \ (\frac{1}{2}) < \frac{1}{2}$ and $\frac{d\hat{} \ (\frac{1}{2})}{dv_l} > 0$. To show $\frac{d\hat{} \ (\frac{1}{2})}{dv_l} > 0$, take derivative w.r.t v_l for both sides of $\frac{d \ (\ ; v_{hb}(\ ))}{d} j_{\ = \frac{1}{2}} = 0$, we have $\frac{dv_{hb}(\frac{1}{2})}{dv_l} = \hat{} \ (\frac{1}{2})$ $1 < 0$.

Case 2: When $_0$ stays in the third partition, $_0 \in [\bar{}(\frac{k}{}), \bar{}^L]$, the EL constraint implies $v_{hb} \quad \frac{k}{}$. Thus $(\, , v_{hb})$ is convex in . Hence, the optimal stopping the seller would like to induce is either $\hat{}(v_{hb})$ (Free Return) or $_0$ (Stochastic return with null learning). The later one is strictly dominated by Learning Deterrence. Hence the optimal selling mechanism is either one of those feasible Free Return mechanism (i.e., $\{v_h \quad v_{hb}, (0, 0)\}$ subject to the EL constraint) or Learning Deterrence.

Case 3: Suppose that belongs to the second partition, $_0 \in [\underline{}, \bar{}(\frac{k}{}))$. Conditional on Lemma 3, (1) for a fixed $v_{hb} \in [\frac{k}{_0}, \frac{k}{})$, the optimal stopping belief would be either $\hat{}(v_{hb})$ or $_0$; (2) for a fixed $v_{hb} \in [\frac{k}{}, v_{hb}(_0)]$, there exists a locally optimal stopping belief (v_{hb}), which gives the seller profit $(\, (v_{hb}), v_{hb})$. In particular, this stopping belief (v_{hb}) corresponds to a locally optimal Stochastic Return mechanism, $\{v_h \quad v_{hb}, (x_r(\, (v_{hb}), v_{hb}), t_r(\, (v_{hb}), v_{hb}))\}$, that is better than any other stochastic return mechanism given v_{hb}. To show Theorem 1, we only need to focus on the the scenario (2). Since (v_{hb}) is monotone, we can consider $(\, , v_{hb}(\,))$ instead of $(\, (v_{hb}), v_{hb})$.[48] Recall that,

$$(\, , v_{hb}(\,)) = \frac{_0}{1}(v_h \quad v_{hb}(\,)) + \frac{1 \quad _0}{1} t_r(\, , v_{hb}(\,))$$

One can easily see this from Figure 4: the seller's profit is a weighted average between price $v_h \quad v_{hb}(\,)$ and the transfer $t_r(\,)$, with weight $\frac{_0}{1}$ and $1 \quad \frac{_0}{1}$ respectively. Since is the solution of $\frac{\partial (\, , v_{hb}(\,))}{\partial} = 0$, the affine curve crossing $(1, v_h \quad v_{hb}(\,))$ and $t_r(\,)$ is tangent to $t_r(\, , v_{hb}(\,))$ at the point $(\, , t_r(\,))$. Hence,

$$(\, , v_{hb}(\,)) = t_r(\, , v_{hb}(\,)) + \frac{\partial t_r(\, , v_{hb}(\,))}{\partial}(_0 \quad)$$

Notice that $\frac{\partial t_r(\, , v_{hb})}{\partial}$ does not depend on v_{hb}, hence by taking derivative w.r.t , we have:

$$\frac{d (\, , v_{hb}(\,))}{d} = \frac{dt_r}{d} \quad \frac{\partial t_r}{\partial} + (_0 \quad)\frac{\partial^2 t_r}{\partial ^2} = \frac{\partial t_r}{\partial v_{hb}}\frac{dv_{hb}}{d} + (_0 \quad)\frac{\partial^2 t_r}{\partial ^2} = \left[\frac{\frac{\partial t_r}{\partial v_{hb}}\frac{dv_{hb}}{d}}{\frac{\partial^2 t_r}{\partial ^2}} + _0 \quad \right]\frac{\partial^2 t_r}{\partial ^2}$$
$$= \left[\frac{(1 \quad)(\frac{k}{v_{hb}(\,)}(v_h \quad v_l) + v_l)}{v_h} + _0 \quad \right]\frac{\partial^2 t_r}{\partial ^2} \qquad (10)$$

It would be useful to mention that the support of $(\, , v_{hb}(\,))$ is $\in [\underline{}, 0.5]$ and hence $\frac{\partial^2 t_r}{\partial ^2} = \frac{k[(2 \quad 1)E(v| \,) \quad (1 \quad)v_l]}{(1 \quad)^2 \, ^3(v_h \quad v_l)} < 0$. Therefore for each $_0$, the monotonicity of $(\, , v_{hb}(\,))$ can be pin down by the sign of $_0 \qquad (\,)$, where $(\,) = \frac{(1 \quad)}{v_h} E(v|\frac{k}{v_{hb}(\,)})$. In particular, if $_0 \quad > (\,)$, $(\, , v_{hb}(\,))$ is decreasing in , otherwise, it's increasing in .

[48] Initially, we consider $\max_{v_{hb}} (\, (v_{hb}), v_{hb})$, which is actually equivalent with considering $\max (\, , v_{hb}(\,))$.

Given Claim 2, we are going to prove the situation that $^{0}(\_\ ) < \ 1$.[49] Denote i as the solution of $^{0}(\ ) = \ 1$, $^{y} = i + \ (i)$ and $^{z} = \_ \ + \ (\_\ )$. Easy to see $^{y} < \ ^{z}$. There are three sub-cases. (a). When $_{0} 2 [\_\ ; \ ^{y}]$; (b) When $_{0} 2 (\ ^{y}; \ ^{z}_{0}]$; (c) When $_{0} 2 (\ ^{z}_{0}; \bar{\ }(\frac{k}{\_}))$.

(a). When $_{0} 2 [\_\ ; \ ^{y}]$, $_{0}$ $(\ )$ for all $_{0}$. Hence $(\ ; v_{hb}(\ ))$ is increasing in . Notice that here $< 0{:}5$,[50] ignoring the *encourage-learning* constraint but considering the Bayesian-plausible constraint, the seller gets the largest profit by inducing stopping at $\min f \ _{0}; 0{:}5g$. If $_{0}$ $0{:}5$, inducing buyer-stopping at prior is the best, which is already shown to be worse than Learning Deterrence. If $_{0} > 0{:}5$, then inducing buyer-stopping at $0{:}5$ and choosing $v_{hb}(0{:}5) = v_{hb}(0{:}5)$ is the best. $(0{:}5; v_{hb}(0{:}5))$ is then a weighted average between $t_{r}(0{:}5) = t_{r}(0{:}5)$ and $t(0{:}5) = v_{h} \ \ v_{hb}(0{:}5)$, which is clearly smaller than Learning Deterrence profit $t(\ _{0})$, since t is increasing and $t(\ _{0}) > t(0{:}5)$.

(b) When $_{0} 2 (\ ^{y}; \ ^{z}]$, the affine curve $_{0}$ crosses $(\ )$ at least once and at most twice. The first crossing belief $i_{1}(\ _{0})$ is towards the left of i and the possible second crossing belief $i_{2}(\ _{0})$ is to the right of i. Precisely, $i_{1}(\ _{0}) = f \ : \ (\ ) = \ _{0} \ ; \ _{0} 2 (\ ^{y}; \ ^{z}]; \ ^{0}(\ ) < \ 1g$ and $i_{2}(\ _{0}) = f \ : \ (\ ) = \ _{0} \ ; \ _{0} 2 (\ ^{y}; \ ^{z}]; \ ^{0}(\ ) > \ 1g$. This implies that $(\ ; v_{hb}(\ ))$ is (weakly) increasing before $< i_{1}(\ _{0})$ and then is quasi-convex for $> i_{1}(\ _{0})$,[51] while if $_{0} = \ ^{z}$, $i_{1}(\ _{0}) = _\ $, $(\ ; v_{hb}(\ ))$ is decreasing first and then possibly increasing. Meaning that the maximum of $(\ ; v_{hb}(\ ))$ is reached either locally at $i_{1}(\ _{0})$ or at the boundary point $\min f \ _{0}; 0{:}5g$. With the same reason as in (a), the boundary solution is dominated by Learning Deterrence.

To show $(i_{1}(\ _{0}); v_{hb}(i_{1}(\ _{0})))$ $t(\ _{0})$, recall that

$$(\ ; v_{hb}(\ )) = \frac{\ _{0}}{1}(v_{h} \ \ v_{hb}(\ )) + \frac{1 \ \ _{0}}{1}t_{r}(\ ; v_{hb}(\ ))$$

Therefore to induce fixed stopping belief with $v_{hb}(\ )$, the seller gets larger expected profit if $_{0}$ is larger. In particular, if $_{0}$ is larger, there is larger probability that a good news arrives and the seller can achieve successful sale. Also, note that $i_{1}(\ _{0})$ is decreasing in $_{0}$. Since we need to control $_{0}$, to abuse the notation a little, we write seller's profit as $(\ ; v_{hb}(\ ); \ _{0})$. Hence, for $_{0} < \ ^{z}$, we have

$$(i_{1}(\ _{0}); v_{hb}(i_{1}(\ _{0})); \ _{0}) < \ (i_{1}(\ _{0}); v_{hb}(i_{1}(\ _{0})); \ ^{z}) < \ (i_{1}(\ ^{z}); v_{hb}(i_{1}(\ ^{z})); \ ^{z})$$

Note that $(i_{1}(\ ^{z}); v_{hb}(i_{1}(\ ^{z})); \ ^{z}) = \ (\_\ ; v_{hb}(\_\ ); \ ^{z})$. Substituting $^{z} \ \_ \ = \ (\_\ )$,[52]

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[49] While if $^{0}(\_\ ) > \ 1$, $(\ ; v_{hb}(\ ))$ is quasi-convex and therefore is maximized at the boundary point $\_$ or $\min f \ _{0}; 0{:}5g$. In the main text, we discuss this scenario.

[50] For larger $_{0}$, inducing stopping at $0{:}5$ and hence choosing $v_{hb}(0{:}5)$ may not satisfy the *encourage-learning* constraint, i.e., $v_{hb}(0{:}5) > v_{hb}(\ _{0})$.

[51] Here quasi-convex could be either decreasing or decreasing and hen increasing.

[52] One can also verify that $\frac{@ \ (\ ; v_{hb})}{@}j_{\_ \ = \frac{k}{v_{hb}}} = 0 =) \ \frac{(1 \ \_\ )(\_\ (v_{h} \ \ v_{l}) + v_{l})}{v_{h}} = \ ^{z} \ \ \_$.

we have,[53]

$$(\_ ; v_{hb}(\_); \ ^{z}) = 0 + (\ ^{z} \quad \_)\frac{@t_r(\ ; v_{hb}(\ ))}{@}j_{=\_} = \frac{k}{v_h}(v_h \quad v_l) + v_l = \mathsf{E}(vj \quad)$$

$\mathsf{E}(vj \quad)$ is smaller than $\mathsf{E}(vj\_^{L})$ since v_h is larger than any v_{hb} that can induce the buyer to learn. Furthermore, $\mathsf{E}(vj\_^{L}) = t(\_^{L}) < t(\ _0)$ for any $\ _0 > \ ^{y}$. Therefore, no locally optimal stochastic return policy can dominate Learning Deterrence.

(c). When $\ _0 \ 2 \ (\ ^{z}; \bar{\ }(\frac{k}{\_}))$, $(\ ; v_{hb}(\ ))$ is quasi-convex, therefore the potential optimal stopping belief is again at the boundaries, either $\_$ or $\min f \ _0; 0{:}5g$. Besides, $\_$ is a inducible (free return) stopping belief for $\ _0 < \bar{\ }(\frac{k}{\_})$, meaning that there exists a Free Return mechanism in which the EL constraint is satisfied that is weakly better than $f v_h \quad \frac{k}{\_}; (0; 0)g$. Furthermore, the boundary solution $\min f \ _0; 0{:}5g$ is strictly dominated by Learning Deterrence. Above all, we show Theorem 1.

Proposition 6. *Denote $\ = \frac{k}{\ }$. If $\ _1 < \ _2$, then $I^{F}(\ _1) \quad I^{F}(\ _2)$.*

This proposition is directly implied by the following Lemma.

Lemma 4. *Denote $\ = \frac{k}{\ }$. $\ ^{F}$ is decreasing in $\ $. t is increasing in $\ $.*

To show that $\ ^{F}$ decreases in $\ $:

$$\frac{@ \ ^{F}}{@} = 1 \quad 2 \ _0 + \frac{p (\ _0 \quad 1) \ _0(v_h \quad 2 \ )}{(\ _0 \quad 1) \ _0(\quad v_h)} < 0$$

(1) When $\ _0 \quad 0{:}5$, this is clear true because $\frac{}{v_h} < 0{:}5$. (2) When $\ _0 < 0{:}5$, $\frac{@\frac{@ \ ^{F}}{@}}{@ \ _0} = 2 + \frac{p(1 \ 2 \ _0)(2 \quad v_h)}{2 \ (1 \quad _0) \ _0 \ (\quad + v_h)} < 0$. Note that $\ _0 > \frac{}{v_h}$ and $\frac{@ \ ^{F}}{@}j_{\ _0 = \frac{}{v_h}} = 0$. Therefore, $\frac{@ \ ^{F}}{@} < 0$ when $\ _0 < 0{:}5$.

To show that $\bar{\bar{t}}(\ _0)$ increases in $\ $. Note that

$$(v_h \quad v_l) + v_l \quad (v_h \quad \bar{v}_{hb}) = \quad + \bar{v}_{hb} \quad (1 \quad) \quad \log(\frac{}{1}) \quad \log(\frac{}{\bar{v}_{hb}})$$

Taking derivative w.r.t $\frac{k}{\ }$ for both sides of the above, we have

$$\begin{aligned}
\frac{d\bar{v}_{hb}}{d} &= 1 + \frac{d\bar{v}_{hb}}{d} \quad (1 \quad) \quad \log(\frac{}{1}) \quad \log(\frac{}{\bar{v}_{hb}}) \quad + \ (1 \quad)\frac{d\log(\frac{}{\bar{v}_{hb}})}{d} \\
&= 1 + \frac{d\bar{v}_{hb}}{d} \quad (1 \quad)\log \frac{=1}{\tilde{\ }=1 \quad \tilde{\ }} + (1 \quad)\frac{\bar{v}_{hb}}{} \frac{\bar{v}_{hb} \qquad (\bar{v}^{0}_{hb} \quad 1)}{(\bar{v}_{hb} \quad)^2} \\
&= 1 + \frac{d\bar{v}_{hb}}{d} \quad (1 \quad)\log \frac{=1}{\tilde{\ }=1 \quad \tilde{\ }} + (1 \quad)\frac{\bar{v}_{hb} \quad \bar{v}^{0}_{hb}}{\bar{v}_{hb}}
\end{aligned}$$

[53] Recall the expression of $\_ = \quad + \sqrt{\quad (\quad + \frac{v_l}{v_h \quad v_l})}$