Information Design in Cheap Talk

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Abstract

This paper discusses strategic communication where the sender can endogenously acquire information and we focus on state-dependent preference. We show that the unrestricted freedom of choosing experiments can align Sender and Receiver's interest and therefore eliminates communication loss. In the benchmark model, the optimal experiment generates conclusive signal (conclusive good news) about the state of one-sided common interest and we pin down the optimal experiment geometrically. When the sender cannot commit to the information structure, it is optimal for him to acquire perfect information.

Keywords: Cheap talk, information design, truthful disclosure, rational inattention, conclusive good news

JEL: D82, D83

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1 Introduction

Starting from Crawford and Sobel (1982), many papers discuss how much a biased expert (Sender) can gain from strategic communication with an uninformed agent (Receiver). However, most of the literature assumes that the expert has perfect information about the state of world. Or equivalently, Sender is presumed to conduct fully-revealing experiment before information transmission. In this paper, we incorporate Sender's optimal choice on information structure at the pre-communication stage and aim to address the following questions. Will it make a difference if Sender (he) can choose what to learn before communicating to the Receiver (she)? Will Sender be less reliable given that he is allowed to acquire information? Does the optimal experiment have some general properties?

Different from Lipnowski and Ravid (2018) and Chakraborty and Harbaugh (2010), which assume Sender's preference is not affected by the state, we focus on state-dependent preference of Sender. It would be useful to shed some light on the reason that we want to study state-dependent utility. First, it is a natural assumption that Sender's preference varies when he has different information. For example, an investor prefers to allocate all his resources to the risky asset if he is sure that the asset is good. Otherwise he prefers to switch to the safety asset. Moreover, it is essential to notice that the degree of alignment or conflict over Sender and Receiver's preference also varies under different states or different beliefs about the state. To see this point, consider the following scenario. Suppose there are two different projects and the manager is uncertain about each one's net present value and the capital intensity. The manager gains larger managerial utility from more capital intensive project, while the shareholder prefers a project that yields higher NPV. Therefore, the manager and shareholder only agree on investing in the same project if one project dominates the other in both attributes. Otherwise, they have conflict over the ideal project if one has higher NPV but less capital intensity. Hence, state-dependent preference implicitly creates one more trade-off for Sender (here refer to the manager). Will he prefer to have more information so that he can make better use of it? If the manager can make his own choice, then he is willing to acquire full information about each project so that he can precisely choose the one inducing the highest managerial utility. However, if the shareholder is the one to make a choice given the information provided by the manager, the value from more information would be offset by the loss from their conflict interests.

Technically, this payoff state-dependence assumption changes the result a lot comparing to cheap talk with transparent motives (Lipnowski and Ravid (2018)), in which

Sender has to be indifferent[1] among different messages since his preference is not affected by his private information. But with state-dependent payoff function, this is not necessarily true and it is possible to have non-binding incentive constraints in communication. By performing the right experiment, Sender can build up credibility of truthful disclosure.

We first show that it is without loss to focus on truth-telling equilibrium in terms of Sender's expected payoff, which indicates no information loss at communication stage. The sufficient statistics that affects Sender's payoff is the total information content delivered to Receiver. Hence, instead of acquiring more information and then garble it through disclosure, he can merely acquire the same net amount of information and disclose them truthfully. This reduction emphasizes the role of information design in cheap talk game: Sender can design the experiment to control how much his interest is aligned with Receiver (Deimen and Szalay (2019)), which endogenously generates commitment power in the communication stage.

As we show in our main result, when Sender has one-sided common interest with Receiver at state 0 (i.e., with binary state space, Sender and Receiver prefer the same action when the state is 0), it is optimal for him to design an experiment that generates conclusive information about state 0. This contradicts with Kamenica and Gentzkow (2011), where the optimal experiment generates inconclusive news about the state of common interest. To illustrate this result, consider binary state space and state-independent payoff function of Sender shown as in figure 1. The number above the solid line is Sender's payoff on Receiver's belief space. Receiver takes three different actions, $\{a_1, a_2, a_3\}$, given different posterior beliefs and she breaks the indifference in Sender's favor. Both Sender and Receiver share common prior belief, $\mu_0 = 0.5$. One can easily check that the optimal experiment under full commitment is $(0.4, 0.8)$. We call it Y experiment. Denote another experiment $(0, 0.8)$ as X experiment. Apparently, Y is better than X since Sender can generate utility 4 more often with Y. However, both experiments are not incentive compatible when Sender cannot commit to reporting the true information outcome. For example, if Sender performs Y experiment when he gets posterior belief 0.8, he will recommend a_1 instead of truthfully recommending a_2.

However, if Sender's payoff becomes state-dependent, his preference over different actions varies when his belief changes. Considering figure 2, suppose when the state is 1, Sender's utility from a_2 remains the same and his utility from a_1 is 2[2]. Then it

[1] The reason is that when Sender's payoff is not affected by what he knows, he has exactly the same preference no matter what his private information is. Hence, if he strictly prefers one message under some belief, he also prefers that message across all other beliefs

[2] As long as Sender's utility of a_2 conditional on state 1 stays in [1.5,2.75], both experiments are

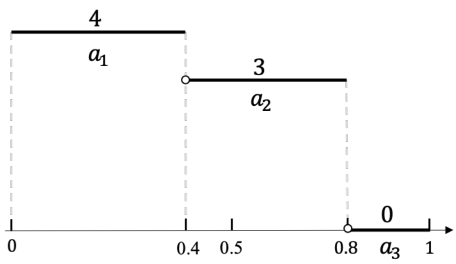

Figure 1: State-independent payoff

is easy to check that both X and Y are incentive compatible for truthful disclosure. For example, when Sender performs X and obtains belief 0.8, he gets utility 3 from reporting the true information outcome and it's larger than the utility $0.8 \times 2 + 0.2 \times 4 = 2.4$ from reporting the unrealized belief 0.4. Remember that X induces a_1 less frequently than Y[3]. Hence Sender is worse off when the true state is 0, but he benefits from getting a_1 less often when the true state is 1. Therefore, the information value comes in. Performing a more informative experiment allows Sender to make better use of the information. In other words, he can make recommendations given more precise belief. In particular, conditional on state being 0, Sender's utility from X is $\frac{3}{4} \times 4 + \frac{1}{4} \times 3 = 3.75$, which is smaller than his utility from Y, $\frac{9}{10} \times 4 + \frac{1}{10} \times 3 = 3.9$. While if the state is 1, Sender's utility from X is 3, larger than his utility from Y, $\frac{3}{5} \times 2 + \frac{2}{5} \times 3 = 1.2$. In total, X outperforms Y. Meanwhile, under the tradition of Blackwell, X is the optimal experiment in this example since it is the most informative one that can credibly induce a_1 and a_2 in the equilibrium. Furthermore, X generates conclusive signal about state 0.

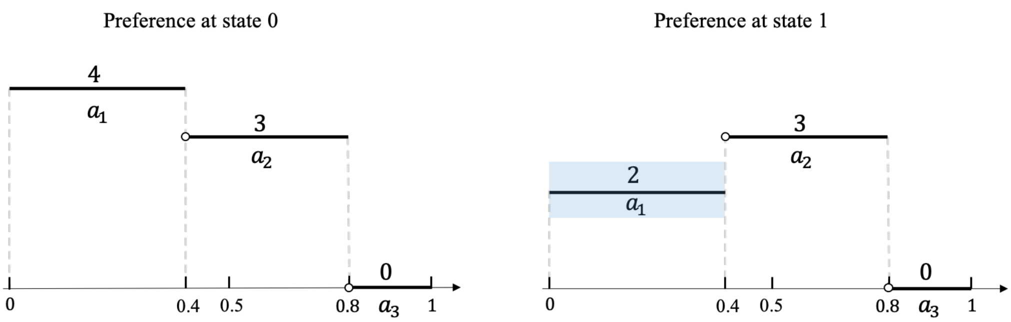

Figure 2: State-dependent payoff

incentive compatible for truthful disclosure.

[3]One can check the signal distribution of these two experiments:

X	0	0.8
state 0	$\frac{3}{4}$	$\frac{1}{4}$
state 1	0	1

Y	0.4	0.8
state 0	$\frac{9}{10}$	$\frac{1}{10}$
state 1	$\frac{3}{5}$	$\frac{2}{5}$

The crucial assumption to ensure fully revealing state 0 being optimal is the one-sided common interest at state 0. Though more information helps Sender to make more precise recommendations, we still need to worry if Receiver's reaction could offset the value add-on from more information. For example, Sender has no incentive[4] to fully reveal state 1, because he gets zero payoff if state 1 is fully revealed to Receiver.

A careful reader would note that in this simple example, experiment X is exactly the KG experiment. However, this is not generally true. Consider the figure 3. Following Kamenica and Gentzkow (2011), $(0.4, 0.8)$ is the optimal experiment under full commitment[5], while if Sender has no commitment power on truthful disclosure, $(0, 0.8)$ is the optimal experiment.

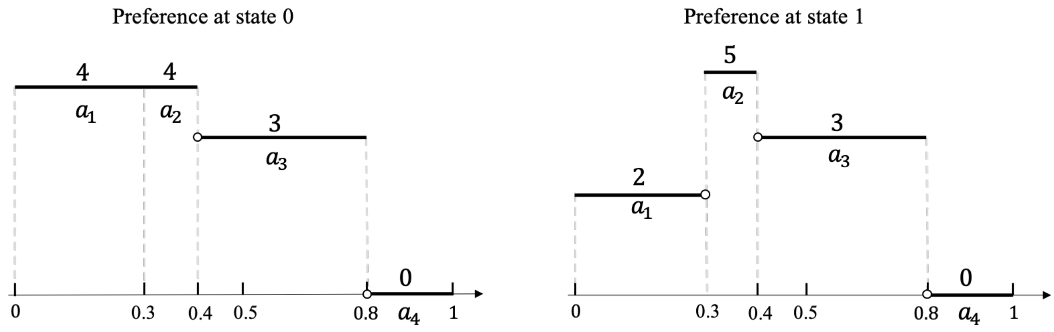

Figure 3: State-dependent payoff

We also discuss the situation that Sender has no commitment power on information design. This case can also be interpreted as that Sender has full control of manipulating the probability of generating different signal realizations. Or the experiment is unobservable to Receiver. Our result indicates that it is without loss to focus on a perfect signal structure as the equilibrium experiment. It rationalizes the assumption in the literature on cheap talk: Sender has perfect information about the true state. Besides, this result coincides with the literature on limited commitment, Sender usually designs a more informative experiment when he has less credibility. However, the mechanism is different in the sense that our result directly comes from the Blackwell theorem, which requires Sender's payoff to be state-dependent. While the literature on limited commitment usually assumes state-independent bias.

Related Literature

There is a group of literature talking about strategic communication with information acquisition. Most of them assume specific utility function and make restrictions on information structure. Argenziano et al. (2016) discuss strategic communication with

[4]Not only because experiments that are fully revealing state 1 is not incentive compatible.

[5]It is not incentive compatible since Sender prefers a_2 over a_3 in both states.

costly information acquisition and they show that equilibrium decisions based on a biased expert's advice may be more precise than when information is directly acquired by the decision-maker. Hence communication outperforms delegation in terms of Receiver's payoff. Deimen and Szalay (2019) have the same result, and they assume the state space is two-dimensional. They find that when Sender can commit to a normal information structure, it is optimal for him to acquire information of equal use to both himself and Receiver, which endogenously align Sender and Receiver's interests. Besides, Kosenko (2018), Strulovici (2017) talk about mediator problems in which after Sender acquires information, the mediator can garble the information.

This paper also stays in the line of Bayesian persuasion (Kamenica and Gentzkow (2011), Dworczak and Martini (2018)) and the latest literature where the full commitment assumption is relaxed. Guo and Shmaya (2018) examines the situation where Sender can costly manipulate the information structure, and the cost is related to the content of distortion. Di Tillio et al. (2017) investigate the persuasion outcome if Researcher is able to use private information to manipulate the experiment, for example, he can use private information to set treatment group and control group. Lipnowski et al. (2018) discuss the situation that Sender has private type on the ability to fake information outcomes. They conclude that Receiver can benefit from a less credible Sender in terms of getting more precise information. Alonso and Camara (2018) study information design problem when tempering information is detectable by Receiver and how Receiver's audit probability affects Sender's choice on experiment. Those papers basically relax the commitment assumption on information design or disclosure policy. My paper discusses two different cases: (1) full commitment on information design and no commitment on disclosure; (2) no commitment on information design and no commitment on disclosure.

This paper mostly relates to Lipnowski and Ravid (2018). They discuss a game where Sender has perfect information of the true state and communicate it to Receiver. They assume transparent motive of Sender and the highest ex-ante payoff Sender can achieve is pinned down by the quasi-concave envelop of the indirect utility function. It is interesting to note that the assumption of a perfectly informed Sender is innocuous: for all feasible equilibrium outcomes, Sender can replicate it by acquiring perfect information at first and then mix over it. However, when Sender's payoff is state-dependent, the freedom of choosing information structure gives Sender strictly positive valuation in many examples.

Furthermore, this paper is very much related to Pei (2015). He discusses a cheap talk game where Sender can costly acquire information which is unobserved by Receiver. In particular, he assumes quadratic loss utility and he restricts the information structure as any finite partitions. The results show that when the cost is sufficiently high, an upwardly biased sender conveys more precise information when recommending larger action, which is not true when the cost is low. In our case, we can have the former result with costless information acquisition. The subtle difference is that Pei (2015) focuses on unobservable information structure, therefore he is emphasizing the incentive constraints (in the information acquisition stage) that Sender does not want to deviate to a more informative information structure when Receiver's decision is fixed. However, in our main part, the incentive constraints of information acquisition stage are relaxed by commitment. Hence, we want to ask what kind of information structure is better, though Receiver's action depends on Sender's choice of information structure. To a certain extent, our last result (given no commitment to experiment) can be interpreted as the limit of Pei (2015) when the cost converges to zero.

The rest of this paper is organized as follows. Section 2 describes the model setting. Section 3 discusses the optimal experiment when Sender can commit to the information structure. Section 4 characterizes the optimal experiment when Sender has no commitment power. Section 5 concludes.

2 Setting

Our model describes a general cheap talk game where Sender (he) can first acquire information and then release information to Receiver (she). In particular, we focus on situations that Sender has state-dependent preferences. Thus, there is a state space with finite states, $\omega \in \Omega$. In the beginning, Sender and Receiver share common prior belief $\mu_0 \in \Delta\Omega$. The game contains two stages. At the first stage, Sender designs an information structure and conducts the experiment with no cost. The experiment or the information structure $\{\pi(\cdot|\omega)\}_{\omega\in\Omega}$ is a distribution of signal realization $s \in \mathcal{S}$ over state. In the main part of this paper, we discuss the case that Sender can commit to π, but we will also have a small section talking about the situation when Sender has no commitment power on π. At the second stage, after observing the experiment outcomes, he sends a cheap talk message $m \in M$ to Receiver which may convey information about the information outcomes he gets. The disclosure policy can be view as an "information structure", $\{\delta(\cdot|s)\}_{s\in\mathcal{S}}$, which is the distribution of message over signal realization space. Note that δ is defined the same way as a garbling matrix in Blackwell's tradition. Receiver then observes the message and decides which action to take, $a \in A$.

Sender's utility $u : \Delta A \times \Omega \to \mathbb{R}$, which depends on the state of the world. Receiver's utility $u^r : A \times \Omega \to \mathbb{R}$. After Sender chooses π and observes the signal realization, he chooses a disclosure policy $\delta : \Pi \times \mathcal{S} \to \Delta M$. Receiver then chooses a decision rule

$\rho : \Pi \times M \to \Delta A$. The belief system[6] $\mu^r : \Pi \times M \to \Delta\Omega$. Hence, an equilibrium consists of three maps such that:

1. $\delta(\pi, s)$ is supported on $\max_{m \in M} E_\omega[u(\rho(\pi, m), \omega)|\mu^s(\pi, s)]$, where $\mu^s(\pi, s)$ is Sender's posterior belief after observing signal s.

2. $\rho(\pi, m)$ is supported on $\max_{a \in A} E_\omega[u^r(a, \omega)|\mu^r(\pi, m)]$.

3. $\mu^r(\pi, m)$ is obtained by μ_0, given experiment π and Sender's disclosure policy δ, using Bayesian rule whenever possible.

The message space can be polished to be $\Delta\Omega$. Hence, an equilibrium can also be described using a recommending system. First, Sender observes a signal and chooses a recommendation from the belief space, $m \in \Delta\Omega$. He can also mix over different recommendations. Receiver then believes Sender's recommendation about the belief she should have and then chooses an action to take. Then belief consistent constraint requires that the recommended belief is exactly the same with the belief that is obtained by Bayesian rule[7], $m \equiv \mu^r(\pi, m)$.

In the whole paper, we focus on Sender preferred PBE. In particular, Sender chooses the equilibrium that gives him the highest ex-ante payoff. We use π^* to represent the optimal information structure. To simplify exposition, we omit the notation for π when it does not cause misunderstanding. Denote $v(\mu^s, \alpha(\mu^r))$ as Sender's expected payoff when his own posterior belief is μ^s and Receiver's belief is μ^r.

$$v(\mu^s, \alpha(\mu^r)) := E_\omega[u(\alpha(\mu^r), \omega)|\mu^s]$$
$$\text{s.t.} \quad \alpha(\mu^r) \in \Delta(\arg\max_a E_\omega[u^r(a, \omega)|\mu^r])$$

Note that when Receiver is indifferent between two actions, she may need to use mixed strategy to ensure existence of informative equilibrium (Lipnowski and Ravid (2018)). Besides, when Sender's payoff is state-dependent, the order of Sender's preference over two different actions may vary. Therefore when we say Receiver breaks the indifference in Sender's favor, denoted as $\alpha^*(\mu^r)$, we formally mean:

$$\alpha^*(\mu) = \arg\max_{\alpha(\mu)} v(\mu, \alpha(\mu))$$

This is defined the same way as in Kamenica and Gentzkow (2011). When Receiver holds the same belief with Sender and breaks the tie in Sender's favor, Sender's expected payoff equals to:

$$\hat{v}(\mu) := v(\mu, \alpha^*(\mu))$$

[6] After Sender observes the signal realization, he updates a posterior belief. However, in an equilibrium, the belief consistent constraint is usually for Receiver.

[7] It is without loss since in equilibrium, Receiver can always back out the right belief given Sender's equilibrium strategies.

One simple observation is that when Sender's preference is perfectly aligned with Receiver, i.e. $\mu^s \in \arg\max_{\mu^r} v(\mu^s, \alpha^*(\mu^r))$, the optimal information structure is the same with KG 2011. This is trivial since Sender has no incentive to misreport his private information, which restores his credibility. We are interested in the optimal information design for a biased Sender.

Same with the standard literature, we are going to use belief-based approach. An experiment π is equivalent to a distribution of posterior belief, which is denoted as $F(\cdot)$. The support of F is the collection of Sender's posterior belief $\mu^s(s)$ after observing each signal $s \in \mathcal{S}$. The disclosure policy Sender uses is to garble the information structure F. Therefore, Receiver forms belief given Sender's disclosure policy:

$$\mu^r(m) = \sum_{\mathcal{S}} \frac{Pr(s)\delta(m|s)}{\sum_{\mathcal{S}} Pr(s)\delta(m|s)} \mu^s(s) \tag{BC}$$

Where $Pr(s) = \sum_{\Omega} \pi(s|\omega)$ is the ex-ante probability that signal s is generated. $\delta(m|s)$ is the chance that Sender reports m when he observes signal s. Apparently, Receiver's posterior belief is a weighted average over Sender's posterior beliefs. Denote $G(\cdot)$ as the distribution of Receiver's posterior belief. Hence, F is a mean-preserving spread of G or G is a garbling of F. In other words, the maximum information Sender can transmit to Receiver is bounded by F.

The belief-based approach allows us to focus on the equilibrium outcomes. Formally, an equilibrium outcome is a triple, $(F, G, v) \in \Delta\Delta\Omega \times \Delta\Delta\Omega \times \mathbb{R}$. This is more complex[8] than Lipnowski and Ravid (2018). Even with binary state space, we need a three dimension graph to represent the feasible equilibrium outcomes. Luckily, Lemma 1 implies that we can narrow down the equilibrium outcome from (F, G, v) to (G, v), as long as (G, v)[9] is induced by an equilibrium.

Claim 1. *If Sender can chooses a disclosure policy δ, Receiver forms posterior belief according to BC, then Sender's expected payoff can be directly pinned down by the distribution of Receiver's posterior belief G.*

$$EU(\pi, \delta, \alpha) = \sum_{\mathcal{S}} Pr(s) \sum_{supp(G)} \delta(\mu^r|s) v(\mu^s(s), \alpha(\mu^r)) = \sum_{supp(G)} Pr(\mu^r) v(\mu^r, \alpha(\mu^r)) \tag{1}$$

Since Sender's expected payoff is linear in his belief for any fixed Receiver's action and μ^r is a weighted average over $\mu^s(s)$ according to BC, there is no difference between summing by signals ($\mathcal{S}$) and summing by messages ($supp(G)$). Furthermore, Sender's ex-ante payoff is not directly affected by the information structure he chooses. In other

[8]In Lipnowski and Ravid (2018), an equilibrium outcome is a pair $(G, v) \in \Delta\Delta\Omega \times \mathbb{R}$. Since with transparent motive, Sender's expected payoff does not depend on his private information. Besides, Sender knows the true state.

[9]Note that for (F, G, v), v is calculated by $v(\mu^s, \alpha(\mu^r))$. For (G, v), v is obtained by $v(\mu, \alpha(\mu))$.

words, Sender can get same ex-ante payoff by choosing different experiments as long as the total information content of both experiment F and disclosure δ, which is G, is the same. If Claim 1 allows us to focus on equilibrium outcome as a pair of (G, v), which is indirectly affected by F through Sender's incentive constraints. Lemma 1 indicates that it is without loss (in terms of Sender's payoff) to focus on equilibrium outcome as a pair of (F, v).

Definition: If for some equilibrium such that $F = G$, we call it a *truth-telling equilibrium.* Otherwise, we call it a *pooling equilibrium.*

Lemma 1. *If for some information structure F, there exists a pooling equilibrium such that Receiver's posterior following G, then with a new information structure $F' = G$, Sender can achieve the same expected payoff by a truth-telling equilibrium.*

To see this, we want to show that the new experiment G is incentive compatible for truthful disclosure. Considering the original pooling equilibrium, each belief μ_j in the support of G is a weighted average over those beliefs $\{\mu_i\}_{i\in\mathcal{I}}$ in the support of F such that if $i \in \mathcal{I}$, type μ_i Sender reports μ_j with positive probability. With revealed preference, type μ_i Sender weakly prefers to report μ_j. Since Sender's expected payoff is linear in his belief, he weakly prefers to reports μ_j when his posterior belief is μ_j. This is true for all $\mu_j \in supp(G)$. Thus, experiment G is credible for a truth-telling equilibrium. Given Claim 1, by performing experiment G and disclosing the true information outcomes, Sender achieves the same ex-ante payoff.

Note that this Lemma does not depend on whether Sender's preference varies over states. From which, one can see the significance of information design in cheap talk game: Sender can choose experiment to align his interest with Receiver, so that he can truthfully report the information outcome he obtains. Communication loss is eliminated by the freedom of information acquisition.

Now, Sender's optimization problem can be replaced to:

$$\sup_{F,\alpha} \; E_F[v(\mu, \alpha(\mu))]$$

$$\text{s.t.} \qquad \text{F is Bayesian Plausible} \tag{BP}$$

$$v(\mu, \alpha(\mu)) \geq v(\mu, \alpha(\mu')), \quad \forall \mu, \mu' \in supp(F) \tag{IC}$$

In the next part, we are going to focus on a special case that Sender is biased toward a certain state and try to find the general property of the optimal information structure.

3 Optimal Experiment for biased Sender

In this section, we are going to focus on binary state space, $\omega \in \{0, 1\}$ and we use μ to denote the probability that state is 1. Besides, we want to discuss a particular group of preferences: Sender has one-sided common interest with Receiver at some state.

Definition: Sender has *one-sided common interest* with Receiver at state 0 if

$$v(0, \alpha(0)) \geq v(0, \alpha(\mu^r)),\ \forall \mu^r \neq 0$$

Sender has *one-sided common interest* with Receiver at state 1 if

$$v(1, \alpha(1)) \geq v(1, \alpha(\mu^r)),\ \forall \mu^r \neq 1$$

WLOG, we focus on the case that Sender and Receiver have one-sided common interest at state 0. The result is symmetric if they have common interest at state 1. To understand this definition, when the true state is 0, among the whole feasible set of Receiver's action, Sender's optimal action is the same with Receiver's. In other words, there is no conflict of interest at state 0. For example, both the investor and financial advisor prefer to direct all resource to a risky asset if it's indeed profitable. A doctor would suggest conservative treatment if the patient's situation is relatively good, while the patient prefers less invasive treatment as well. The shareholder and manager prefers the same project if one dominates the other in terms of both NPV and capital intensity. Similar examples are easy to generate. One may also note that this one-sided common interest can be implied by downward bias. Suppose Receiver's ideal action equal to the expected state, while Sender has quadratic loss utility and is biased to action lower than the expected state. Hence Sender and Receiver both prefers 0 action when the true state is 0.

Note that when we assume one-sided common interest at state 0, we implicitly allow for conflict interest at state 1 (i.e., Sender and Receiver prefers different actions when the true state is 1). Since we are using belief-based approach, one can consider common interest (same preferred action) as that given Sender's private belief, he prefers Receiver to hold the same belief with him. While if there is conflict of interest, Sender prefers Receiver to hold a different belief comparing to his private belief. To make the result tractable, we makes several technical assumptions on Sender's preference.

Assumption 1. *$u(\alpha^*(\mu^r), \omega)$ is supermodular.*

Assumption 2. *$\inf_\alpha v(\mu^s, \alpha(\mu^r))$ and $\sup_\alpha v(\mu^s, \alpha(\mu^r))$ are quasi-concave in μ^r for $\mu^s \in \{0, 1\}$.*

Assumption 3. *$\sup_\alpha v(\mu^s, \alpha(\mu^r))$ is upper-semi continuous in μ^r and $\inf_\alpha v(\mu^s, \alpha(\mu^r))$ is lower-semi continuous in μ^r. Both have finitely many one-side jumps.*

Assumption 1 is to ensure the existence of an equilibrium in the second stage. Think about an extreme case that Sender's utility is not supermodular: he prefers high μ^r when his own posterior belief is low while he prefers low μ^r when his posterior is high. Then a truth-telling equilibrium will not exist, since he has incentive to over-report the information when his posterior belief is low while under-report the information when his posterior belief is high. Given Assumption 2, we can show that $\inf_\alpha v(\mu^s, \alpha(\mu^r))$ and $\sup_\alpha v(\mu^s, \alpha(\mu^r))$ are quasi-concave in μ^r for all $\mu^s \in [0, 1]$. This assumption is to simplify the analysis so that Sender's preference $v(\mu^s, \alpha(\cdot))$ is well-ordered over μ^r for all feasible Receiver's decision rule. Assumption 3 is just some regularity conditions (Dworczak and Martini (2018)).

Proposition 1. *(Conclusive good news) When A2, A3 hold and Sender and Receiver have one-sided common interest at state 0, if there exists an informative equilibrium experiment, then it is optimal for Sender to design an experiment that generates conclusive news about state 0.*

$$\pi^*(s = 0|\omega = 0) > 0, \ \pi^*(s = 1|\omega = 1) = 1$$

Proof. In the below long proof, I first show that the optimal experiment can be restricted to binary signal structure. Second, I show that for any given experiment $F_Y = (\mu_1, \mu_2)$, if there exist a truth-telling equilibrium, we can always construct a new experiment $F_X = (0, \mu_2)$ without violating Sender's incentive constraints. Finally, I prove that the new experiment F_X dominates the original experiment F_Y in terms of Sender's ex-ante payoff. Hence, it is without loss (in terms of Sender's expected payoff) to focus on experiments in which $\mu(s = 0) = 0$.

Claim 2. *For any F such that $|supp(F)| > 2$, if there exists a truth-telling equilibrium with $G = F$, then there exists another truth-telling equilibrium with F' such that $|supp(F')| = 2$ that gives sender weakly higher ex-ante payoff.*

The idea for this claim is simple. As long as there exists a truth-telling equilibrium with more than two poster beliefs, Sender can always use two of them, that has a (weakly) higher conditional expected payoff, as a new experiment, and the incentive constraints still hold.

The second step is straightforward given A2, A3 and one-sided common interest at state 0:

$$v(\mu_2, \alpha(\mu_2)) \geq v(\mu_2, \alpha(\mu_1)) \geq v(\mu_2, \alpha(0)) \tag{2}$$

where the first inequality comes from that the original experiment is incentive compatible. The second inequality comes from A2 and A3, which is formally proved in the appendix[10]. Besides, $v(0, \alpha(0)) > v(0, \alpha(\mu_2))$ is merely the definition of one-sided common interest.

Third step, we need to show F_X dominates F_Y in terms of Sender's ex-ante payoff. To simplify expositions, we use y_k to denote signal in experiment F_Y and x_j to denote signal in F_X. Given the particular way that we construct F_X, we can write down a garbling matrix[11] B such that $F_X B = F_Y$. Each element b_{jk} represents the probability of generating y_k when observing x_j, therefore $\sum_k b_{jk} = 1$.

$$\begin{array}{cc} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \end{array} & \begin{pmatrix} b_{11} = 1 & b_{12} = 0 \\ b_{21} > 0 & b_{22} > 0 \end{pmatrix} \end{array} \tag{B}$$

Note that x_2 and y_2 generate the same posterior belief μ_2, which means that y_2 only contains information from x_2. Hence, $b_{12} = 0$. Besides, both x_1 and x_2 generate y_1 with strictly positive probability. Therefore, the information contained in y_1 is a weighted average of x_1 and x_2. This is a standard result of any garbling:

$$Pr(\omega_n|y_k) = \sum_j c_{jk} Pr(\omega_n|x_j) \tag{3}$$

Where $c_{jk} = b_{jk}\frac{Pr(x_j)}{Pr(y_k)}$. The posterior belief after observing y_k is a weighted average[12] over the posterior beliefs of x_j such that $b_{jk} > 0$. Hence, Sender's ex-ante payoff of observing y_1 given Receiver's belief being $\mu(y_1)$ is:

$$\begin{aligned} v(\mu(y_1), \alpha(\mu(y_1))) &= v(\sum_j c_{j1}\mu(x_j), \alpha(\mu(y_1))) \\ &= c_{11}v(\mu(x_1), \alpha(\mu(y_1))) + c_{21}v(\mu(x_2), \alpha(\mu(y_1))) \\ &= c_{11}v(0, \alpha(\mu_1)) + c_{21}v(\mu_2, \alpha(\mu_1)) \\ &\leq c_{11}v(0, \alpha(0)) + c_{21}v(\mu_2, \alpha(\mu_2)) \\ &= \sum_j c_{j1}v(\mu(x_j), \alpha(\mu(x_j))) \end{aligned}$$

The inequality comes from two driven forces: (1) Sender and Receiver have one-sided common interest at state 0; (2) Revealed preference: in the original equilibrium where $F_Y = (\mu_1, \mu_2)$, type μ_2 Sender prefers to truthfully report μ_2, which implies

[10] Basically, we show that if A2 holds, then Sender's expected payoff is quasi-concave for all feasible $\alpha(\mu^r)$.

[11] To calculate the garbling matrix, we need to refer to the original signal structure $\{\pi_X(\cdot|\omega)\}_{\omega\in\Omega}$ and $\{\pi_Y(\cdot|\omega)\}_{\omega\in\Omega}$.

[12] $Pr(\omega_n)Pr(y_k|\omega_n) = \sum_j b_{jk}Pr(x_j|\omega_n)Pr(\omega_n)$ and divide both sides by $Pr(y_k)$, we have the above equation (3).

$v(\mu_2, \alpha(\mu_2)) \geq v(\mu_2, \alpha(\mu_1))$. Furthermore, Sender's ex-ante payoff for observing y_2 is:

$$\begin{aligned} v(\mu(y_2), \alpha(\mu(y_2))) &= v(\sum_j c_{j2}\mu(x_j), \alpha(\mu(y_2))) \\ &= c_{12}v(\mu(x_1), \alpha(\mu(y_2))) + c_{22}v(\mu(x_2), \alpha(\mu(y_2))) \\ &= c_{12}v(0, \alpha(\mu_2)) + c_{22}v(\mu_2, \alpha(\mu_2)) \\ &= \sum_j c_{j2}v(\mu(x_j), \alpha(\mu(x_j))) \end{aligned}$$

The last equality holds because $c_{12} = b_{12}\frac{Pr(x_1)}{Pr(y_2)} = 0$ and $\mu(x_2) = \mu(y_2)$. It is important to ensure $\mu(x_2) = \mu(y_2)$ which is equivalent to letting $b_{12} = 0$. Since otherwise, if $b_{12} > 0$, then $\mu(x_2) > \mu(y_2)$ and $v(\mu(x_2), \alpha(\mu(y_2)))$ and $v(\mu(x_2), \alpha(\mu(x_2)))$ are not well-ordered. With our construction of F_X and F_Y, the following inequality holds.

$$v(\mu(y_k), \alpha(\mu(y_k))) \leq \sum_j c_{jk}v(\mu(x_j), \alpha(\mu(x_j))),\ \forall k$$

Multiplying both sides by $Pr(y_k)$ and summing over k, we have Sender's expected payoff of choosing F_X weakly higher than F_Y.

$$\sum_k Pr(y_k)v(\mu(y_k), \alpha(\mu(y_k))) \leq \sum_j Pr(x_j)v(\mu(x_j), \alpha(\mu(x_j)))$$

□

This propositions implies that when Sender and Receiver have one-sided common interest at state 0 (state 1), he will optimally design an experiment such that state 0 (state 1) would be fully revealed given some realization. One can interpret it as conclusive good news. This contradicts with Kamenica and Gentzkow (2011), since the optimal experiment generates inconclusive news about the state of common interest. Furthermore, recall that one-sided common interest at state 0 can be implied by downward bias. Therefore, if Sender has consistent incentive to under-report the information, then the optimal experiment generates precise information about the low state. This is also different from Crawford and Sobel (1982), where Receiver process low message as much noisier information(less reliable) as Sender always has incentive to under-report the true state.

In Blackwell theorem, agent's value function is convex in belief since the action space is exogenous. Therefore, acquiring more information allows the agent to make more precise decision. This is not generally true in strategic environment. The major problem is that Receiver's action depends on Sender's choice of information structure. Therefore, a more informative experiment could make Sender worse off if Receiver takes extremely non-preferred action when more information is revealed. Hence, while picking up the optimal choice of information structure, Sender faces a tension between acquiring more information and alleviating the conflict of interest. On the one hand, Sender can take advantage of additional information by making more precise recommendations (information value). On the other hand, the information structure is used to control how much the two parties' interest is aligned, or it could be coarsely geared to reduce conflict of interest.

To see this clearly, consider the figure 4. Loosely speaking, the way we construct F_X is to split $\mu(y_1)$ into a pair of more spread beliefs, $\mu(x_1)$ and $\mu(x_2)$. When Sender performs F_Y and obtains signal with belief $\mu(y_1) = \mu_1$, Receiver takes $\alpha(\mu_1)$. By splitting y_1 into x_1 and x_2, Sender can incentivize Receiver to take $\alpha(\mu(x_2))$, which is a better choice for Sender under his posterior belief $\mu(x_2)$, as shown by the blue arrow. In other words, having more information, Sender can make more precise recommendation in stead of pooling $\mu(x_1)$ and $\mu(x_2)$ together to make one single recommendation. Hence, there will be a weakly positive payoff by obtaining more information, fixing Receiver's action space. Moreover, when Sender indeed conducts F_X, Receiver's best response to a new information structure also changes. In particular, she takes $\alpha(0)$ when $\mu(x_1) = 0$ is revealed. Fortunately, as shown by the green arrow, this is exactly the action that Sender preferred when he confirms that the true state is 0. Therefore, by performing an experiment that generates conclusive news about state 0, Sender obtains an additional gain from common interest at state 0.

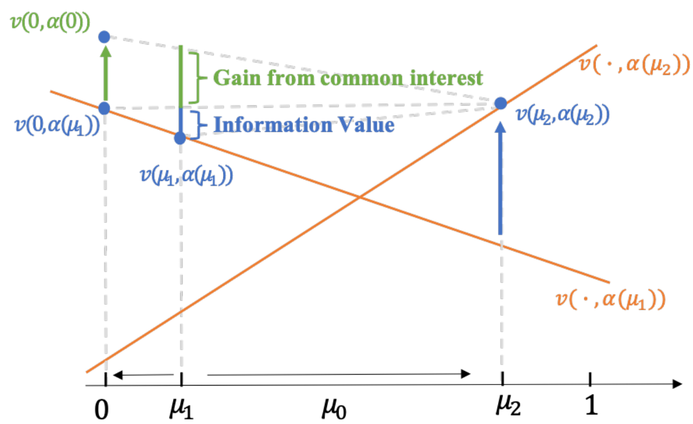

Figure 4: Conclusive good news

A symmetric argument, however, will not hold for conclusive news about state 1. For example, if fixing μ_1 and splitting μ_2 into μ_1 and 1. Sender will still be better off in terms of information value: conditional on the splitting, he can recommend a better action $\alpha(\mu_1)$ in stead of inducing $\alpha(\mu_2)$. On the other hand, when state 1 is fully revealed, there will be a loss from conflict interest at state 1, $v(1, \alpha(1)) << v(1, \alpha(\mu_2))$. In total, experiment that fully reveals state 1 could be suboptimal. Furthermore, note that this proposition only implies that experiment $(0, \mu_2)$ is better than (μ_1, μ_2). It is not necessarily true that any (μ_1', μ_2) with $\mu_1' \in (0, \mu_1)$ dominates (μ_1, μ_2). In other words, Sender's payoff is not monotone when having more information about state 0. With stronger assumption, like downward bias, this monotonicity can be generated, shown in the appendix. Intuitively, when Sender has a consistent incentive to under-represent the information, he wants to reduce Receiver's belief. However the credible way to reduce Receiver's belief is to design experiment that makes s_1 more conclusive about the state 0.

3.1 Optimal experiment

After we prove the optimal experiment is conclusive about the state 0, we can pin down the optimal experiment using a geometric approach. We assume that Receiver breaks the tie in Sender's favor, which can be verified with out loss of generality after we pin down the result. Given A1[13] and proposition 1, Sender's incentive constraint with $F = (\mu_1, \mu_2)$ can be reduced to the inequality below.

$$\mu_2 \geq \frac{1}{1 - \frac{u(\alpha^*(\mu_2),1) - u(\alpha^*(0),1)}{u(\alpha^*(\mu_2),0) - u(\alpha^*(0),0)}} \tag{4}$$

Note that both sides are function of μ_2. Denote B as the solution of inequality (4), which is a set of μ_2. B is not necessarily to be convex. Now we construct a new function: $\overline{v}(\mu) : [0, \max\{B\}] \to \mathbb{R}$. Note that the set $[0, \max\{B\}]$ is the union of three non-overlap sets, $0 \cup B \cup O$, where $O = \cup_{n \in \mathcal{N}} O_n$ is a collection of open sets and $O_n \cap O_{n'} = \emptyset$. Denote $\underline{O}_n$ as the left limit of O_n, $\overline{O}_n$ as the right limit of O_n.

$$\begin{cases} \overline{v}(\mu) = \hat{v}(\mu), & \text{if } \mu = 0 \text{ or } \mu \in B \\ \overline{v}(\mu) \text{ is affine and} & \\ \overline{v}(\underline{O}_n) = \hat{v}(\underline{O}_n), \overline{v}(\overline{O}_n) = \hat{v}(\overline{O}_n), & \text{if } \mu \in O_n, \forall n. \end{cases} \tag{5}$$

Therefore, we can construct a new function $\overline{V}(\cdot)$ as the concavification of $\overline{v}(\cdot)$.

$$\overline{V}(\mu) = \sup_{\mu', \mu'' \in [0, \max B],\ \gamma \in [0,1],\ s.t\ \gamma\mu' + (1-\gamma)\mu'' = \mu} \{\gamma \overline{v}(\mu') + (1-\gamma)\overline{v}(\mu'')\}$$

Proposition 2. *(Optimal Experiment) With A1, A2 and A3, the highest ex-ante payoff sender can achieve is $\overline{V}(\mu_0)$, whose solution $(0, \mu_2^*)$ is the optimal information structure as long as $\mu_2^* \in B$.*

[13] A1 is stronger than the sufficient condition to ensure a solution of inequality (4).

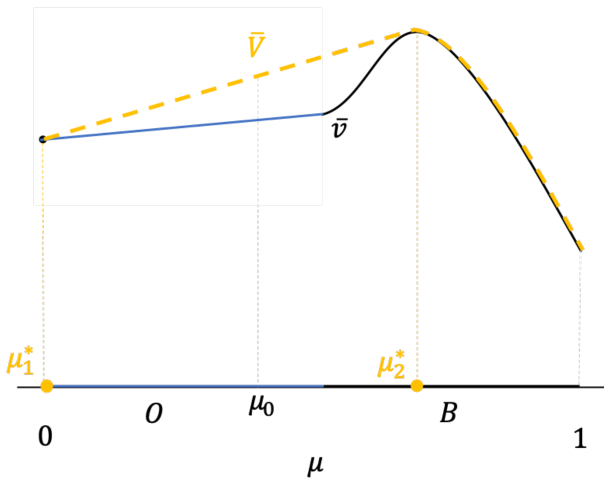

Figure 5: Optimal experiment

Figure 5 is an example showing how we construct $\overline{v}$ and $\overline{V}$. First, we find the set of μ_2 such that Sender weakly prefers to report the true information outcomes, then $\overline{v}$ remains the same with $\hat{v}$ in this set. Second, $\overline{v}$ remains the same with $\hat{v}$ when $\mu_1 = 0$. Third, we draw an affine curve connecting the two endpoints for each compact subset of $(0, \max\{B\}]/B$. Last, we construct the concavifivation of $\overline{v}$. The way we construct $\overline{v}$ is to ensure the optimal solution can locate within the region that Sender's incentive constraints hold. Besides, it is possible that there exists multiple information structures that are optimal for Sender.

As a conclusion, the algorithm to pin down the optimal experiment is the following. (1) First, we check which state Sender and Receiver have common interest. If state 1 is of common interest, then we fix $\mu_2 = 1$. Otherwise, we fix $\mu_1 = 0$. (2) Second, we construct $\overline{v}$ and the concave envelope $\overline{V}$ with the same method as above. Then we can pin down the optimal experiment.

3.2 Finite States

In this section, we assume that Receiver's action only depends on the expected state. Given that the optimal experiment is conclusive about the state of common interest under binary state space, a natural conjecture would be that Sender optimally chooses an experiment that fully reveals all states that of common interest. However, this is not necessarily true. For example, if $\Omega = \{1, 2, 3, 4\}$ and state 3 is of common interest. Then an experiment that fully reveals state 3 may not be incentive compatible for truthful disclosure. What we can show is that if Sender and Receiver has common interest at the lowest state (highest state), then the optimal experiment generates conclusive information about the lowest state (highest state). Sorry that we abuse our notation a little bit here, we use μ^r to represent the expected state of Receiver given her posterior belief. Hence, A1-3 remain the same.

Proposition 3. *(Conclusive news about the lowest state) If A2, A3 hold, Sender and Receiver have common interest at the lowest state, then $\pi^*_{n1} = 0$ for all $n > 1$ if an optimal experiment exists.*

4 When Sender Cannot Commit

In this section, we would like to analyze a different game such that Sender has no commitment power on both information structure and disclosure policy. In particular, after Sender proposes an experiment to Receiver, he can change the underlying probability of generating each signal realization with no constriction. This is an extreme case of limited commitment. In other words, Sender has the least credibility. We find it is interesting since the experiment itself becomes part of the equilibrium. In particular, there is one more incentive constraint to check - Sender does not want to deviate to another experiment.

Lemma 2. *For any finite state space, if Sender cannot commit to the information structure, then an (informative) truth-telling equilibrium exists if and only if there exists an experiment π and some $\alpha(\cdot)$ such that for each n,*

$$u(\alpha(\mu(s_i)), \omega_n) = \max_{s \in \mathcal{S}} u(\alpha(\mu(s)), \omega_n) \text{ for all } i \text{ such that } \pi_{ni} > 0 \tag{6}$$

When Sender has no commitment power on information structure, a truth-telling equilibrium requires two-stage incentive constraints. (1) In stage 1: given Receiver's decision rule (a mapping from message to action) in the communication stage, Sender weakly prefers to perform the experiment that he proposes to Receiver. (2) In stage 2: given the information structure Sender chooses, he weakly prefers to report the true information outcomes, which is same as previous sections. The first one is important when the proposed experiment is not perfectly informative. If that is the case, deviating to a more informative experiment allows for (weakly) better use of the information in terms of which message to send. Condition 6 is to ensure that a more informative experiment does not give Sender strictly positive information value. Otherwise, the incentive constraint in stage 1 fails. Furthermore, condition 6 itself is strong enough to ensure the incentive constraint in stage 2.

To understand condition 6, consider an arbitrary experiment π shown as the above matrix. The raw represents states $\{\omega_1, \omega_2, \omega_3\}$ and the column represents signals

$\{s_1, s_2, s_3\}$.

$$\begin{pmatrix} \pi_{11} & 0 & 0 \\ 0 & \pi_{22} & \pi_{23} \\ \pi_{31} & 0 & \pi_{33} \end{pmatrix}$$

If this experiment is an equilibrium experiment, then when the true state is ω_2, Sender has to be indifferent between reporting s_2 and s_3. Besides, both are optimal for Sender when he knows the true state is ω_2. In other words, for each state ω_n and for all s_i that is generated with positive probability in ω_n, Sender has to be indifferent among reporting those signals. In addition, reporting those signals is optimal for Sender. If this is not true, then Sender is strictly better off by deviating to a perfect information structure. To see this, if reporting s_3 is not optimal for Sender when ω_2 is fully revealed, he is strictly better off by splitting s_3 into two signals such that ω_2 and ω_3 are fully revealed separately. Then when ω_2 is revealed, he can report the signal that gives him higher payoff than s_3. This is strictly better than having non-degenerate belief $\mu(s_3)$ and reporting s_3 to Receiver. Hence, deviating to a more informative experiment gives Sender strictly positive valuation.

Lemma 3. *If there exists a pooling equilibrium such that Sender performs experiment F and Receiver's inference follows G, then there exists a truth-telling equilibrium such that Sender performs experiment G.*

For Sender's incentive constraints in the second stage (communication stage), the proof is exactly the same with the proof of proposition 1. Here, we need to focus on Sender's incentive constraint for not deviating to other experiments. Since experiment F leads to a pooling equilibrium, Sender weakly prefers to performs F rather than some more informative experiments. Then for the experiment F, if state ω_n generates more than two realizations, Sender has to be indifferent between those realizations, which means that in ω_n, he is indifferent among all the messages that he is going to disclose given those realizations. Otherwise, he strictly prefers to deviate to perfect signal structure[14]. Thus, we can write down the analogous condition of condition 6.

$$\forall n,\ u(\alpha(\mu_j), \omega_n) = \max_{supp(G)} u(\alpha(\mu_{j'}), \omega_n), \forall \mu_j \in supp(G), \mu_j(\omega_n) > 0$$

Note that $\mu_j(\omega_n) > 0$ is equavalent with $\pi_{nj} > 0$. Therefore, given lemma 2, if Sender performs experiment G in the first place and truthfully discloses the outcomes, then he will not deviate to other experiments because Jensen's inequality binds.

Proposition 4. *In terms of Sender's payoff, it is without loss to focus on Sender performing perfect signal structure. An informative equilibrium exists if and only if there exists a distribution G such that for some $\alpha(\cdot)$ and all n,*

$$u(\alpha(\mu_j), \omega_n) = \max_{supp(G)} u(\alpha(\mu), \omega_n), \forall \mu_j \in supp(G), \mu_j(\omega_n) > 0 \tag{NC}$$

[14]The argument is analogous to the proof of lemma 2.

Given lemma 2 and 3, we can focus on truth-telling equilibrium. Hence, if Sender is doing experiment F in the equilibrium, he get exactly the same payoff by doing a perfect signal and using F as the disclosure policy: In particular, if π_{ni} is the probability that s_i is generated by state ω_n in the experiment F, then with a perfect signal structure, Sender discloses $\mu(s_i)$ with probability π_{ni} when he gets a perfect signal of ω_n.

Note that with no commitment, the condition to ensure equilibrium is stronger. To see this, for a fixed distribution of Receiver's inference G, Sender who has commitment power on information structure, can try different experiments so that his posterior belief would be different and it is possible that there exists an experiment such that Sender's incentive constraints hold. However, this is not true when he has no commitment power. Condition (NC) is fixed whenever G is fixed, which means that as long as G is not incentive compatible given F to be a perfect signal structure, G cannot be incentive compatible for any other information structure. Furthermore, proposition 4 kind of rationalizes the assumption of cheap talk literature that Sender has perfect information at the beginning. It also kind of coincides with many papers discussing limited commitment: a less credible Sender designs a more informative experiment. However, our result directly comes from Blackwell theorem and has little correlation with Sender's bias.

In the last, commitment on experiment gives Sender non-negative valuation. Since with no commitment power on experiment, Sender weakly better off by performing a perfect signal. Then the equilibrium set of no commitment must be a subset of the equilibrium set with commitment: when Sender has commitment power on information structure, he can also choose to perform a perfect signal and the no commitment equilibrium is still available. Hence, Sender weakly better off by having commitment power on information structure.

5 Conclusion

This paper discusses cheap talk game where Sender can endogenously acquire information with no restriction. We find that the freedom of choosing experiment allows we to focus on truth-telling equilibrium. This is different from the nature of cheap talk game where communication loss incurs due to conflict of interests. While in our game, Sender can choose experiment in order to align his interest with Receiver. Furthermore, we focus on situation where Sender's payoff is state-dependent. We find that when Sender and Receiver have one-sided common interest at the lowest, it is optimal for him to design a information structure that fully reveals the lowest state, which is commonly viewed as conclusive good news. This result is driven by both information value and the gain from common interest. In the last, we also talk about situation when the information structure is not observable, under which if there is no equilibrium for a perfectly informed Sender, then there exists no equilibrium. This result rationalizes the assumption in cheap talk literature that Sender has perfect information about the true state.

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6 Appendix

Proof of Claim 1

For simplicity of exposition, we omit the notion of α and we want to show:

$$EU(\pi,\delta,\alpha)=\sum_{\mathcal{S}} Pr(s)\sum_{supp(G)}\delta(m|s)v(\mu^s(s),\mu^r(m))=\sum_{supp(G)}Pr(m)v(\mu^r(m),\mu^r(m))$$

Recall that we let $\mu^r(m)=m$, hence the above equation can be simplified as

$$\sum_{\mathcal{S}} Pr(s)\sum_{supp(G)}\delta(\mu^r|s)v(\mu^s(s),\mu^r)=\sum_{supp(G)}Pr(\mu^r)v(\mu^r,\mu^r)$$

Note that $Pr(\mu^r)=\sum_{\mathcal{S}} Pr(s)\delta(\mu^r|s)$ is the ex-ante probability of obtain recommendation μ^r, which can be directly obtained from $G(\cdot)$. We prove this equation from RHS to the LHS.

$$\begin{aligned}
\sum_{supp(G)}Pr(\mu^r)v(\mu^r,\mu^r) &= \sum_{supp(G)}Pr(\mu^r)\sum_{\Omega}\mu^r u(\mu^r,\omega)\\
&= \sum_{supp(G)}Pr(\mu^r)\sum_{\Omega}\sum_{\mathcal{S}}\frac{Pr(s)\delta(\mu^r|s)}{Pr(\mu^r)}\mu^s(s)u(\mu^r,\omega)\\
&= \sum_{supp(G)}\sum_{\Omega}\sum_{\mathcal{S}}Pr(s)\delta(\mu^r|s)\mu^s(s)u(\mu^r,\omega)\\
&= \sum_{\mathcal{S}}Pr(s)\sum_{supp(G)}\delta(\mu^r|s)\sum_{\Omega}\mu^s(s)u(\mu^r,\omega)\\
&= \sum_{\mathcal{S}}Pr(s)\sum_{supp(G)}\delta(\mu^r|s)v(\mu^s(s),\mu^r)
\end{aligned}$$

Proof of Lemma 1

Denote $supp(F)=\{\mu^s(s_1)...\mu^s(s_I)\}$ and $supp(G)=\{\mu^r(m_1)...\mu^r(m_J)\}$. Then we can construct a new experiment $F'=G$. Hence $|\mathcal{S}'|=|supp(G)|$ and $\mathcal{S}'=\{s'_1...s'_J\}$. More specifically, for all $\mu^r(m_j)$ such that $m_j\in M$, we assign $\mu^s(s'_j)=\mu^r(m_j)$, where $Pr(s'_j)=\sum_{\mathcal{S}} Pr(s_i)\delta(m_j|s_i)$.

Assuming there exists a truth-telling equilibrium with the new experiment F', then Receiver's inference is the same, $G'=F'=G$, and the message space does not change. Now we check if the incentive constraints for the truthful report hold: in the original equilibrium, for each $m_j\in M=supp(G)$, there exists a non-empty set $\mathcal{I}$ such that $\delta(m_j|s)>0$ for all $s\in\mathcal{I}$, which indicates that

$$v(\mu^s(s),\alpha(\mu^r(m_j)))\geq v(\mu^s(s),\alpha(\mu^r(m_{j'}))),\quad \forall m_j,m_{j'}\in M \text{ and } m_{j'}\neq m_j$$

Note that $\mu^s(s'_j) = \mu^r(m_j) = \sum_{\mathcal{I}} \frac{Pr(s)\delta(m_j|s)}{\sum_{\mathcal{I}} Pr(s)\delta(m_j|s)} \mu^s(s)$, which is a weighted average over Sender's posterior beliefs of $s \in \mathcal{I}$. It is important that Sender's expected payoff is linear in his belief:

$$v(\mu^s(s'_j), \alpha(\mu^r(m_j))) = \sum_{\mathcal{I}} \frac{Pr(s)\delta(m_j|s)}{\sum_{\mathcal{I}} Pr(s)\delta(m_j|s)} v(\mu^s(s), \alpha(\mu^r(m_j)))$$

Therefore, if for all $s \in \mathcal{I}$, type $\mu^s(s)$ Sender weakly prefer to report m_j, then type $\mu^s(s'_j)$ Sender weakly prefers to report m_j. Hence, a truth-telling equilibrium exists and distribution of Receiver's inference is the same with the original equilibrium. Given lemma 1, Sender's expected payoff remains the same.

Proof of equation (2)

Denote $\sup_\alpha v(\mu^s, \alpha(\mu^r)) := \overline{f}(\mu^r)$ and $\inf_\alpha v(\mu^s, \alpha(\mu^r)) := \underline{f}(\mu^r)$, where we omit the notion for μ^s. Denote $f(\mu^r) = \alpha(\mu^r)\overline{f}(\mu^r) + (1 - \alpha(\mu^r))\underline{f}(\mu^r)$. We want to show that

$$f(\lambda\mu_1^r + (1-\lambda)\mu_2^r) \geq \min\{\overline{f}(\mu_1^r), \overline{f}(\mu_2^r)\} \tag{7}$$

Suppose it is not true, then $f(\lambda\mu_1^r + (1-\lambda)\mu_2^r) < \overline{f}(\mu_1^r)$ and $f(\lambda\mu_1^r + (1-\lambda)\mu_2^r) < \overline{f}(\mu_2^r)$. Since, $f(\lambda\mu_1^r + (1-\lambda)\mu_2^r) \geq \underline{f}(\lambda\mu_1^r + (1-\lambda)\mu_2^r)$, then by semi-continuity and payoff function is one-side jump, there exists a $\mu_3^r \to \lambda\mu_1^r + (1-\lambda)\mu_2^r$ such that μ_3^r is a convex combination of $\mu_1^r + (1-\lambda)\mu_2^r$ and μ_2^r. Then $\overline{f}(\mu_3^r) < \overline{f}(\mu_2^r)$ and $\overline{f}(\mu_3^r) < \overline{f}(\mu_1^r)$, which violates A2.

Since inequality (7) is true for all $\alpha(\mu^r)$, then $v(\mu^s, \alpha(\mu^r))$ is quasi-concave in μ^r no matter what the mixing probability Receiver is using.

Proof of Claim 2

To see this, consider a truth-telling equilibrium with an experiment F that consists I signal realizations, $\{s_1, s_2 ... s_I\}$. Denote Sender's payoff of getting each signal realization as v_i. When state space is binary, we can draw a graph, in which each point of $(\mu(s_i), v_i)$ represents the feasible information outcome and the corresponding payoff. Next, we can connect the any two points with an affine curve in the graph and find the concavification, which is exactly a piecewise linear function. Besides, each end point is an outcome pair. Given the concavification and the prior belief, we can pin down the two end points (should be Bayesian plausible) that gives Sender weakly higher payoff than any combinations of the end points. Suppose the two end points is s_1 and s_2. Then the new experiment $(\mu(s_1), \mu(s_2))$ weakly dominates the original experiment. In addition, the new experiment is incentive compatible given the original experiment with more possible posterior beliefs is incentive compatible.

Monotone comparative statics:

Denote $\mu^*(\mu^s) = \arg\max_{\mu^r} \min_\alpha v(\mu^s, \alpha(\mu^r))$, which represents the set of Sender's most preferred Receiver's belief (given his private information) when Receiver treats him the worst.

Definition 2: Suppose that $\inf \mu^*(\mu^s) \leq \mu^s$ for all $\mu^s \in [0,1]$ and if $\inf \mu^*(\mu^s) = \mu^s$, then $\mu^s \in \mu^*(\mu^s)$. We say that Sender has a *downward bias*[15].

Corollary 1. *When A2, A3 hold Sender has downward bias, if experiment* (μ_1, μ_2) *is incentive compatible, then Sender weakly prefers another experiment* (μ_1', μ_2) *for all* $\mu_1' < \mu_1$.

Proof of Proposition 2

To prove this, we need two steps. In the first step, we assume that Receiver breaks the tie in Sender's favor and show that solution of $\overline{V}(\mu_0)$ is the optimal information structure. In the second step, we show that the assumption on Receiver's action is without loss.

First step. Let $(\overline{\mu}_1, \overline{\mu}_2)$ be a solution of $\overline{V}(\mu_0)$. For each $\overline{\mu}_2$, there could be a set of $\overline{\mu}_1$, denoted as $\mathcal{U}(\overline{\mu}_2)$. With our construction of $\overline{v}$ and $\overline{V}$, it is obvious that $\mu_1^s = 0 \in \mathcal{U}(\overline{\mu}_2)$ for all feasible $\overline{\mu}_2$, since $\overline{V}(\cdot)$ is affine on $[0, \mu_0]$. Then denote $\mathcal{U}(0)$ as the set of $\overline{\mu}_2$ when $\overline{\mu}_1 = 0$. Apparently, if there exist a $\mu_2^s \in \mathcal{U}(0)$ and stays in the set of B, we are done. If there exists a $\mu_2^s \in \mathcal{U}(0)$ and this $\mu_2^s \notin B$, then there exits another ${\mu^s}'_2 \in B$ and is the limit point of the open set that μ_2^s belongs to.

Second step. We want to check if we can without loss assume that Receiver breaks the tie to Sender's favor. To show this, we only need the belief space that is incentive compatible does not shrink if we assume $\alpha = \alpha^*$. If we let $\alpha(0) = \alpha^*(0)$, then by A2 and A3: (1) Sender has the highest ex-ante payoff when he knows the true state is 0, hence he will not report any other messages; (2) When Sender obtains belief μ_2^s, he has the least incentive to send message 0. If $\alpha(\mu^r)$ such that $\mu^r > \mu_0$ is supported on a set with more than one element, then there are only two possibilities:

(1) $\mu^r = \mu_2^s$ is on the part that Sender's payoff $v(\mu_2^s, \alpha(\cdot))$ is weakly increasing. Then Sender's incentive constraint hold no matter what the order of $v(\mu_2^s, \alpha^*(\mu_2^s))$ and $v(\mu_2^s, \alpha(\mu_2^s))$ is. Hence, assuming that Receiver breaks the tie in terms of Sender's preference does not affect Sender's incentive compatibility given experiment $(0, \mu_2^s)$.

(2) $\mu^r = \mu_2^s$ is on the part that Sender's payoff $v(\mu_2^s, \alpha(\cdot))$ is weakly decreasing. Then there is only one possibility that $v(\mu_2^s, \alpha^*(\mu_2^s)) \geq v(\mu_2^s, \alpha(\mu_2^s))$ and $v(0, \alpha^*(\mu_2^s)) \geq v(0, \alpha(\mu_2^s))$. Otherwise, $alpha^*$ is not in Sender's favor. Then if Sender's incentive

[15] Suppose that $\sup \mu^*(\mu^s) \geq \mu^s$ for all $\mu^s \in [0,1]$ and if $\sup \mu^*(\mu^s) = \mu^s$, then $\mu^s \in \mu^*(\mu^s)$. We say Sender has an *upward bias*.

constraint holds for α, it holds for α^*. We are done.

Proof of Proposition 3

$\Omega = \{\omega_1, \omega_2 ... \omega_N\}$. $\omega_n \in \mathbb{R}_+$ and higher n denotes higher state. Suppose there exists a truth-telling equilibrium with experiment F_Y ($\mathcal{S}_Y = \{y_1 ... y_K\}$), then we are going to show that an experiment F_X ($\mathcal{S}_X = \{x_1 ... x_J\}$), such that $x_{n1} = 0$ if $n > 1$, dominates F_Y in terms of Sender's ex-ante payoff if there exists a matrix B such that $F_X B = F_Y$. Note that b_{jk} represents the probability of signal y_k is generated given signal x_j. Besides, $b_{jk} = 0$ if $k \neq j$ for all $k > 1$, which implies that $\mu(y_k) = \mu(s_j)$ if $j = k > 1$. WLOG, we can assume $K = J = N$.

$$\begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1J} \\ 0 & x_{22} & \cdots & x_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & x_{N2} & \cdots & x_{NJ} \end{pmatrix} \begin{pmatrix} b_{11} & 0 & \cdots & 0 \\ b_{21} & b_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ b_{J1} & 0 & \cdots & b_{JK} \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1K} \\ y_{21} & y_{22} & \cdots & y_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ y_{N1} & y_{N2} & \cdots & y_{NK} \end{pmatrix}$$

Step 1, we need to show that the new experiment F_X is incentive compatible for a truth-telling equilibrium. Since F_Y is incentive compatible and $\mu(y_k) = \mu(s_j)$ if $j = k > 1$, Sender's incentive constraints for all realizations x_j such that $j > 1$ still hold. Note that $\mu(\omega_1|x_1) = 1$, hence $E(\omega|x_1)$ is the smallest feasible expectation. Given A2 and A3, $v(\mu(x_1), \mu(x_1)) \geq v(\mu(x_1), \mu(x_j))$ for all $j > 1$. Besides, $E(\omega|x_1) < E(\omega|y_1)$ implies that $v(\mu(x_j), \mu(x_j)) = v(\mu(y_k), \mu(y_k)) \geq v(\mu(y_k), \mu(y_1)) \geq v(\mu(x_j), \mu(x_1))$ for all $j = k > 1$.

Step 2, we need to show that F_X dominates F_Y in terms of Sender's payoff. Similar with the proof of proposition 1, if $v(\mu(y_k), \mu(y_k)) \leq \sum_j c_{jk} v(\mu(x_j), \mu(x_j))$, then F_X is better than F_Y. For $k = 1$:

$$\begin{aligned} v(\mu(y_1), \mu(y_1)) &= v(\sum_j c_{j1}\ \mu(x_j), \mu(y_1)) \\ &= \sum_j\ c_{j1}\ v(\mu(x_j), \mu(y_1)) \\ &= c_{11}\ v(\mu(x_1), \mu(y_1)) + \sum_{j>1} c_{j1}\ v(\mu(x_j), \mu(y_1)) \\ &\leq c_{11}\ v(\mu(x_1), \mu(x_1)) + \sum_{j>1} c_{j1}\ v(\mu(x_j), \mu(x_j)) \\ &= \sum_j c_{j1}\ v(\mu(x_j), \mu(x_j)) \end{aligned} \tag{8}$$

For $k > 1$, since $\mu(y_k) = \sum_j c_{jk}\ \mu(x_j) = \sum_j b_{jk} \frac{Pr(x_j)}{Pr(y_k)} \mu(x_j) = c_{jk}\mu(x_{j=k})$. Therefore,

$$v(\mu(y_k), \mu(y_k)) \leq \sum_j c_{jk}\ v(\mu(x_j), \mu(x_j)), \forall k > 1 \tag{9}$$

Combining inequality (8) and (9), we restore the same inequality coming from convex value function in the proof of Blackwell theorem. Therefore, a more informative F_X dominates F_Y in terms of Sender's payoff.

Step 3, the existence of matrix B is obvious from the nature of F_X. Note that the posterior belief of x_j is the same with y_k is $j = k > 1$ and x_1 fully reveals state 1.

Proof of lemma 2

Before we discuss the proof, it is necessary to notice that any experiment, that is not a perfect signal, is a garbling of perfect signal. Perfect signal is an identity matrix. And the garbling matrix is exactly the experiment itself. Hence, π_{ni}, which is the probability that s_i is generated when the state is ω_n, can also be treated as the garbling probability that s_i is generated when Sender observes a signal that fully reveals state n.

First, we prove the only if part. Suppose this is not true, then there exist a ω_n and s_i such that $u(\alpha(\mu(s_i)), \omega_n) < \max_{s\in\mathcal{S}} u(\alpha(\mu(s)), \omega_n)$. Then if Sender deviates to a perfect signal structure, for all n such that $\pi_{ni} > 0$,

$$v(\mu(s_i), \alpha(\mu(s_i))) = \sum_n c_{ni}\ u(\alpha(\mu(s_i)), \omega_n) < \sum_n c_{ni}[\max_s u(\alpha(\mu(s)), \omega_n)]$$

Where $c_{ni} = \pi_{ni}\frac{Pr(\omega_n)}{Pr(s_i)}$. Then with the same argument with proposition 1, one can check that Jensen's inequality is slack now. Hence, deviating to a perfect signal structure gives Sender strictly higher payoff. There does not exists an equilibrium.

Second, we prove the if part. Note that $\mu(s_i) = \sum_n c_{ni}\ \mu(\omega_n)$, where $\mu(\omega_n)$ denotes that belief that true state is ω_n. Suppose $u(\alpha(\mu(s_i)), \omega_n) = \max_{s\in\mathcal{S}} u(\alpha(\mu(s)), \omega_n)$ for all i such that $\pi_{ni} > 0$, then

$$v(\mu(s_i), \alpha(\mu(s_i))) = \sum_n c_{ni}\ u(\alpha(\mu(s_i)), \omega_n)$$

Which is better than deviating to report other signals. Hence, the incentive constraints hold.